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OPTIMIZATION OF VEHICLE SUSPENSIONS
SUBJECTED TO RANDOM EXCITATION

by

Rifaquat Ali Cheema

A Thesis Submitted

in

Partial Fulfillment

of the

Requirements for the Degree of

MASTER OF SCIENCE

in

MECHANICAL ENGINEERING

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OPTIMIZATION OF VEHICLE SUSPENSIONS
SUBJECTED TO RANDOM EXCITATIONS

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ABSTRACT

This study provides basic information on the analysis and optimization of vehicle suspension systems with a damped absorber attached to the sprung mass and the unsprung mass. This study is also concerned with comparing these systems with a optimized conventional system.

A two degree of freedom linear model subjected to guideway irregularity, described as a hyperbolic displacement spectral density, random excitation is chosen for studying the system. Dimensionless space and design parameters are selected to allow for adequate generality.

The objective function incorporates the tire-terrain normal force, as an indication of the vehicle controllability, constrained by the sprung mass acceleration as a comfort criteria. Optimum parameter synthesis of damped absorber suspension with the damped absorber attached to sprung and unsprung mass, as well as a conventional suspension system has been obtained. Performance characteristics for the optimum damped absorber suspension and the conventional suspension are presented.

The comparison among the optimised conventional, and the optimized damped absorber suspension systems show that the optimum damped absorber suspension with the absorber attached to the unsprung mass, based on the

objective function chosen, is the best.

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NOMENCLATURE

$a_n(s)$	A polynomial of order (n) for the variable s.
a_0, a_1, \dots, a_{n-1}	Coefficient of polynomial a(s).
A	A constant depends on particular terrain.
$b(s)$	A polynomial of order (n) for the variable s.
b_0, b_1, \dots, b_{n-1}	Coefficient of polynomial b (s).
C_2	Damping coefficient of the damper of the main suspension system.
C_3	Damping coefficient of the damper of the dynamic absorber.
c_{2a}, c_{2b}	Damping coefficient of both dampers of the four degrees of freedom model.
d_0, d_1, \dots, d_n	Coefficients of the polynomial d (s).
$d_n(s)$	A polynomial of order (n) for the variable s.
$D(\emptyset)$	Non dimensional characteristic equation of the two degrees of freedom model.
f_g	Dynamic variation of the ground force.
F_g	Non dimensional variation of the ground force.
$H(\emptyset), H(\gamma)$	Non dimensional transfer function between an output and an input.
I_n	Value of complex time integral with (n) number of poles
K_1	Stiffness rate of tire of the two degrees of freedom model.
K_2	Stiffness of the spring of the main suspension of two degrees of freedom.
K_3	Stiffness of the spring of the dynamic absorber of two degrees of freedom model.

$K_{1a}, K_{1b}, K_{2a}, K_{2b}$	Stiffness of the four springs of the four degrees of freedom model.
K_2, K_3	Non dimensional stiffness rate of the springs of the main suspension and the spring of the absorber, for the two degrees of freedom model.
L	The distance between side (a) and (b) of the four degrees of freedom model.
m_1	Unsprung mass of the two degrees of freedom model.
m_2	Sprung mass of the two degrees of freedom model.
m_3	Mass of the dynamic absorber.
m_{1a}, m_{1b}	Unsprung masses of the four degrees of freedom model.
m_{22}	Sprung mass of the four degrees of freedom model.
M_1	Non dimensional unsprung mass of the two degrees of freedom model.
M_3	Non dimensional absorber mass.
n	Order of polynomial.
p	An intermediate coefficient.
q	An intermediate coefficient.
$R(t)$	Autocorrelation function.
S	Laplace operator.
$S_x(w)$	Input displacement spectral density in the time domain.
$S_x(w_{nn})$	Value of input displacement spectral density at w .
$S_x(\Omega)$	Spectral density of terrain surface in spatial domain.
$S_x(\gamma)$	Input displacement spectral density in the non dimensional time domain.

$S_{\ddot{x}'x}(w)$	Sprung mass acceleration spectral density for an input displacement spectral density.
t	Time.
v	Forward vehicle velocity.
x	Input displacement.
x_1	Displacement of the unsprung mass of the two degrees of freedom model.
\dot{x}_1	Velocity of the unsprung mass of the two degrees of freedom model.
\ddot{x}_1	Acceleration of the unsprung mass of the two degrees of freedom model.
x_2	Displacement of the sprung mass of the two degrees of freedom model.
\dot{x}_2	Velocity of the sprung mass of the two degrees of freedom model.
\ddot{x}_2	Acceleration of the sprung mass of the two degrees of freedom model.
x_0	A significant length which describes the magnitude of the input.
X	Non dimensional input displacement.
X_1	Non dimensional displacement of the unsprung mass of the two degrees of freedom model.
\dot{X}_1	Non dimensional velocity of the unsprung mass of the two degrees of freedom model.
\ddot{X}_1	Non dimensional acceleration of the unsprung mass of the two degrees of freedom model.
X_2	Non dimensional displacement of the sprung mass of the two degrees of freedom model.
\dot{X}_2	Non dimensional velocity of the sprung mass of two degrees of freedom model.
\ddot{X}_2	Non dimensional acceleration of the sprung mass of the two degrees of freedom model.

\ddot{y}_c	Comfort limit on acceleration.
\ddot{y}_c	Non dimensional comfort limit on acceleration.
z_{1a}, z_{1b}	Input displacement of both sides of the four degrees of freedom model.
z_2	Displacement of the sprung mass of the four degrees of freedom model.
γ	Non dimensional frequency.
θ	Angular displacement of the sprung mass of the four degrees of freedom model.
μ	Ratio of mass of absorber to the unsprung mass.
f_2	Non dimensional damping factor of the main suspension of the two degrees of freedom model.
f_3	Non dimensional damping factor of the absorber.
α	A multiple of standard deviation.
τ	Shift or lag of one function with respect to another.
\emptyset	Non dimensional laplace operator
w	Input circular frequency.
w_{n2}	Natural frequency of the sprung mass.
w_{n3}	Design natural frequency of the absorber.
w_{nn}	Design natural frequency of the two degrees of freedom model.
c	Constraint.
max	Maximum
min	Minimum
rms	The root mean square.

\int

Integration sign.

j

PSD

Power spectral density.

1. INTRODUCTION

Vibration isolation concerns means to bring about a reduction in vibratory effect.

A vibration isolator in its most elementary form may be considered as a resilient member connecting the equipment and foundation. The function of an isolator is to reduce the magnitude of motion transmitted from a vibrating foundation to the equipment, or to reduce the magnitude of force transmitted from the equipment to its foundation.

1.1. Mechanical Isolation

The performance criteria in the design of mechanical isolation systems are usually evaluated by any of the following characteristics (4).

i. Absolute Transmissibility

It is a measure of the reduction of transmitted force or motion afforded by an isolator. If the source of vibration is an oscillating motion of the foundation, transmissibility is the ratio of the vibration amplitude of the equipment to the vibration amplitude of the foundation.

If the source of vibration is an oscillating force originating within the equipment, transmissibility is the ratio of the force amplitude transmitted to the foundation to the amplitude of the exciting force.

ii. Relative Transmissibility

It is the ratio of the relative deflection amplitude of the isolator to the displacement amplitude imposed at the foundation. This relative deflection is a measure of the clearance required in the isolator.

iii. Motion Response

It is the ratio of the displacement amplitude of the equipment to the quotient obtained by dividing the excitation force amplitude by the static stiffness of the isolator.

1.2 Vehicle Suspension System

Vehicle suspension synthesis can be considered in part as an application of mechanical isolation theory. A considerable amount of work has been performed on general vehicle suspension analysis, design, and optimization (5, 6, 8, 9, 14, 16, 19, 20, 21, 24, 30). Few studies are available in the field of optimum design-parameter synthesis (6, 8, 19, 20, 21, 30). Very little has been done in the application of damped absorber to

vehicle suspension (3, 14, 24).

Most of the work has been on simple model and specified disturbance. Analysis of vehicle suspension has been largely confined to one-dimensional two degrees of freedom vehicle model (5, 6, 8, 14, 16, 20, 21). Some studies have been concerned with one-dimensional three degrees of freedom model (30), and few have included multi-degree of freedom systems (19).

The simple single degree of freedom vehicle model is usually considered in case of handling new developed investigation such as synthesis of active air cushion suspension (34), optimum linear preview control (8, 9), vehicle suspension minimizing perceived acceleration, and optimum linear vehicle suspension subjected to simultaneous guideway and external force disturbances.

System disturbances are usually considered individually and are caused by guideway irregularities. They cover step input (8, 9, 20, 21, 30), different pulses (19, 20), Sinusoids (14, 24) and random inputs (5, 6, 8, 9, 16, 21, 30, 34). Hedrik (35) and Young (36) have considered simultaneous effect of guideway and external force (wind gust) disturbances.

Systems which have been considered are usually linear except for few studies (5, 19, 21, 24, 30). Non

linear exponential damping and non linear exponential elastic restoring elements (19, 21) have been widely used. Some times coulomb friction and non-linearity due to loss of contact between tire and terrain (5, 30) are considered. Unsymmetric damping has been treated by Thompson (30), while actual tire characteristic has been investigated by Omata (24).

Solution of randomly excited non-linear system using analogue computer have been widely exploited (5, 30). If the excitation can be idealized as a Gaussian white noise, exact and approximate solutions can be used based on Markove-vector approach (37). In case of non white stationary excitation approximate solutions can be obtained by perturbation and/or equivalent linearization technique. Wen (32) has presented an approximate method for the non stationary random vibration of non-linear systems.

Most of the previous studies select a behavior variable as a performance criterion. These selected variables include body acceleration as a comfort criteria and ground force (normal force between tire and terrain) as a controllability criteria. However, in some cases, perceived acceleration has been chosen as a ride comfort criterion (5).

The mean square value of the behavior variable is usually employed when the excitation is random (5, 6, 8, 9, 16, 21, 30). Despite the method of mean square integral evaluated by Beshora (10) the method developed by Phillipi (22) is still commonly used. The peak value of the behavior variables is usually considered for different impulse inputs.

Some studies have included constraints on some additional behavior variables, where the physical limits should not be exceeded. The constrained optimization problem, in this case usually consists of a behavior variables as an objective function constrained by another behavior variable (20, 21). An alternate formulation of such problems is to consider some combination of the individual behavior variables as a single overall objective function (6, 8, 9, 16, 34, 35, 36).

Several studies use trade off between two behavior variables usually body acceleration and rattle space (6, 8, 9, 36), ground force and rattle space or ground force and body acceleration (20, 21). Other studies (19, 21) employed the behavior variables as an unconstrained objective function.

1.3 Scope of Study

In this study a one dimensional two degrees of freedom vehicle model, subjected to guideway disturbances (random) is selected. Vehicle controllability as indicated by the tire-terrain normal force is considered to be a primary criterion, constraint by a comfort criteria namely body acceleration.

The effect of adding an optimum damped absorber to the vehicle suspension on the characteristic performances of the vehicle is studied and the pertinent results are compared with an optimized conventional suspension.

2. PROBLEM FORMULATION AND SOLUTION

2.1 Vehicle Suspension Model

The equivalent diagram of an automobile (Figure 1) shows the individual components which are relevant to vibration investigation. It has ten degrees of freedom, the body has six-three in translational and three in rotation and four degrees of freedom for wheel masses which are shown in individual sprungs.

Since such a large number of degrees of freedom complicates the solution, the model is simplified to a two dimensional (Figure 2), where only heaves and pitching are considered by assuming that roughness elements are equal in the left and right tracks, or by assuming the other translation and rotational motions are not coupled with heaves and pitching oscillations. The later case is a vehicle that is symmetric about the longitudinal axis.

This two dimensional model reduces to one-dimensional if $J_2 = m_{22} L_a L_b$. Figure 3a shows the resulting one dimensional model, where the vehicle body and load represents the sprung mass. The suspension is modeled as massless element providing force between the body and the unsprung mass, which is in turn supported

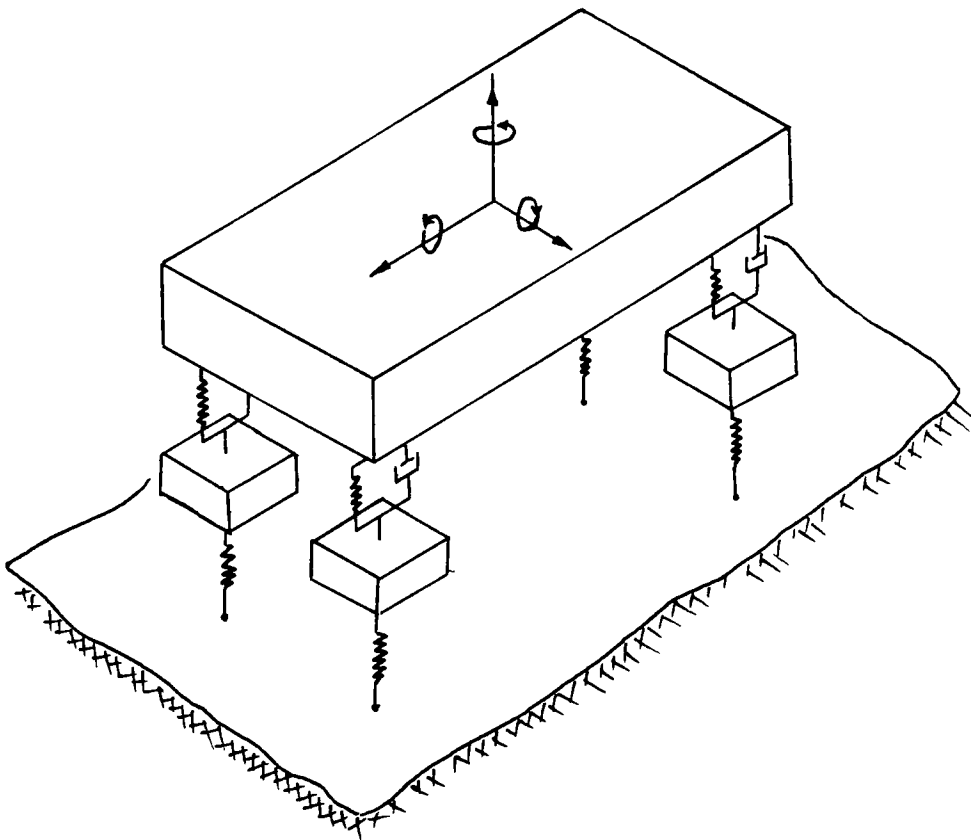


Fig.1. Equivalent diagram of a vehicle

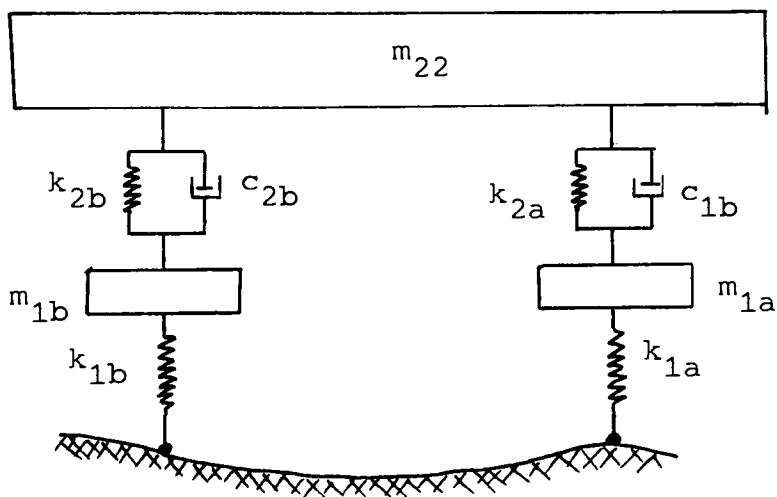


Fig.2. Two dimensional four degrees of freedom vehicle model.

above the roadway by a linear spring. Although this one dimensional vehicle model is very simple, it nonetheless, contains some important features found in the variety of real vehicle suspension system, when designed to decouple the different motions of the vehicle or to handle each motion separately.

Simplified tire characteristic is adopted since the amount of damping in most tires are not effective. If this simplification is not used the effect of change in the inflation pressure and the geometry of rolling wheel, which results in filtering the input, would make the exact representation of the tire characteristic very complex.

In this study the suspension system considered is a passive suspension represented by a linear spring and a linear shock absorber. The damped absorber is represented by a linear mass-spring-dashpot system.

2.2 The Mathematical Models

Three models are considered: the conventional (Figure 3a) model without a damped absorber, with the damped absorber attached to sprung mass (Figure 3b), with the damped absorber attached to unsprung mass (Figure 3c).

2.2.1 Conventional System (without absorber)

Equations of motion for the system shown in Figure 3a are:

$$m_1 \ddot{x}_1 = k_1(x - x_1) - k_2(x_1 - x_2) - c_2(\dot{x}_1 - \dot{x}_2) \text{ ----- (1)}$$

$$m_2 \ddot{x}_2 = k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) \text{ ----- (2)}$$

For non-dimensionalization we consider the following non-dimensional design parameters which will help us in design process. Particularly when keeping m_2 and k_1 constant, then a change in any of M_1 , M_3 , K_2 , K_3 would allow a direct observation of the proportional change in m_1 , m_3 , k_2 and k_3 .

$$M_1 = \frac{m_1}{m_2} \quad M_3 = \frac{m_3}{m_2}$$

$$K_2 = \frac{k_2}{k_1} \quad K_3 = \frac{k_3}{k_1}$$

$$\zeta_2 = \frac{c_2}{2m_2 w_{n2}} \quad \zeta_3 = \frac{c_2}{2m_2 w_{n3}}$$

$$\text{where } w_{n2} = \sqrt{\frac{k_2}{m_2}} \text{ and } w_{n3} = \sqrt{\frac{k_3}{m_2}}$$

$$\text{we define design natural frequency as } w_{nn} = \sqrt{\frac{k_1}{m_2}}$$

$$x_1 = \frac{x_1}{x_0} \quad x_2 = \frac{x_2}{x_0}$$

$$x_3 = \frac{x_3}{x_0} \quad x = \frac{x}{x_0}$$

where x_0 is the length related to the input magnitude.

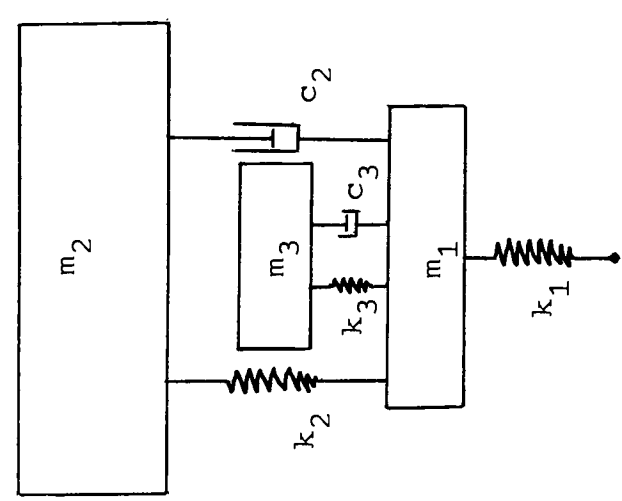
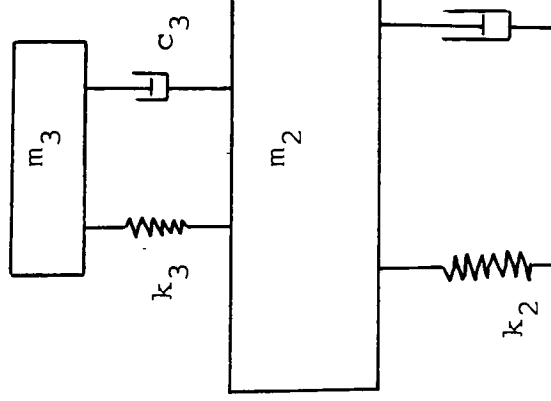
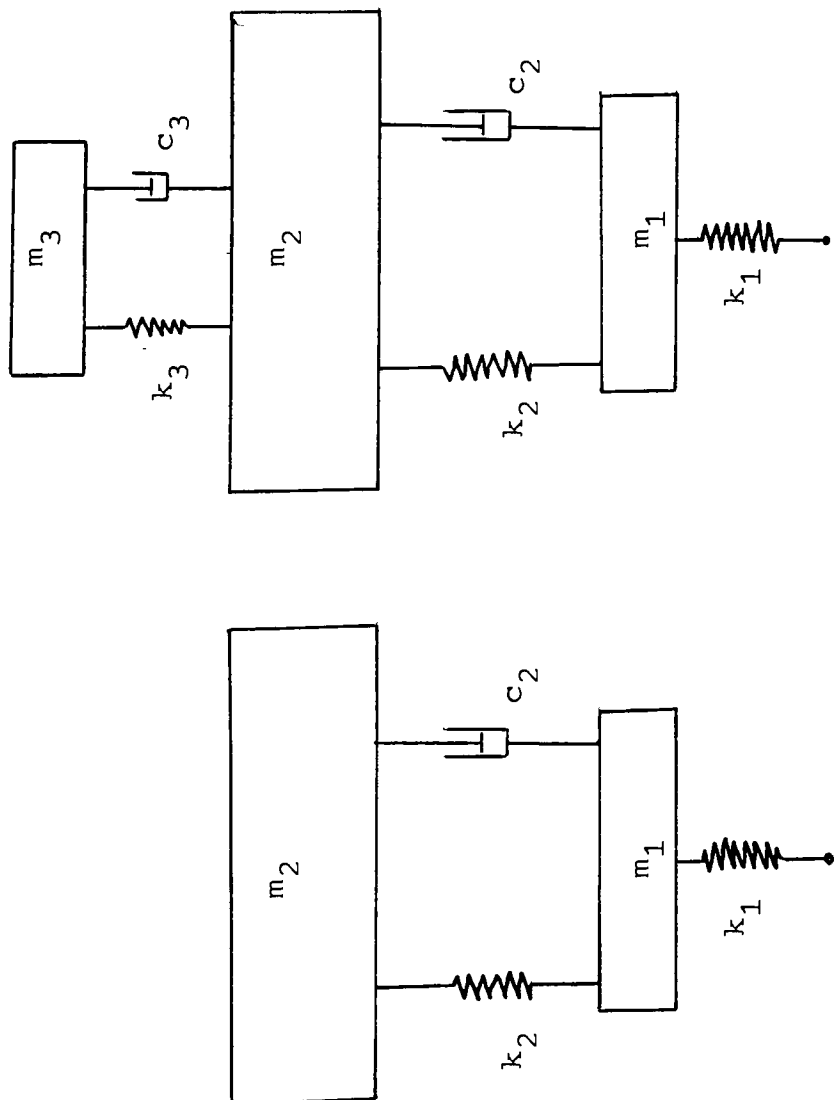


Fig. 3_a. The conventional one dimensional two degree of freedom vehicle model.
 Fig. 3_b. One dimensional three degrees of freedom vehicle model with absorber attached to sprung mass.
 Fig. 3_c. One dimensional three degrees of freedom vehicle model with absorber attached to unsprung mass.

now

$$\dot{x}_1 = \frac{\dot{x}_1}{x_0 w_{nn}} \quad \dot{x}_2 = \frac{\dot{x}_2}{x_0 w_{nn}}$$

$$\dot{x}_3 = \frac{\dot{x}_3}{x_0 w_{nn}}$$

$$\ddot{x}_1 = \frac{\ddot{x}_1}{x_0 w_{nn}^2} \quad \ddot{x}_2 = \frac{\ddot{x}_2}{x_0 w_{nn}^2}$$

$$\ddot{x}_3 = \frac{\ddot{x}_3}{x_0 w_{nn}^2}$$

Substituting these non-dimensional parameters in the equations of motion and with simple mathematical operations.

$$\ddot{x}_1 = \frac{1}{M_1} (x - x_1) - \frac{K_2}{M_1} (x_1 - x_2) - 2 f_2 \sqrt{\frac{K_2}{M_1}} (\dot{x}_1 - \dot{x}_2) \quad \text{-- (3)}$$

$$\ddot{x}_2 = K_2 (x_1 - x_2) + 2 f_2 \sqrt{K_2} (\dot{x}_1 - \dot{x}_2) \quad \text{-- (4)}$$

Taking the laplace transform and writing in the matrix form.

$$\begin{bmatrix} s^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 f_2 \sqrt{\frac{K_2}{M_1}} s & -(\frac{K_2}{M_1} + 2 f_2 \sqrt{\frac{K_2}{M_1}} s) \\ -(K_2 + 2 f_2 \sqrt{K_2} s) & s^2 + K_2 + 2 f_2 \sqrt{K_2} s \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{x}{M_1} \\ 0 \end{bmatrix}$$

Determinant of the matrix

$$(\emptyset^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 \int_2 \frac{\sqrt{K_2}}{M_1} \emptyset) (\emptyset^2 + K_2 + 2 \int_2 \sqrt{K_2} \emptyset)$$

$$- (K_2 + 2 \int_2 \sqrt{K_2} \emptyset) (\frac{K_2}{M_1} + 2 \int_2 \frac{\sqrt{K_2}}{M_1} \emptyset)$$

$$D(\emptyset) = d_4 \emptyset^4 + d_3 \emptyset^3 + d_2 \emptyset^2 + d_1 \emptyset + d_0 \text{ ----- (5)}$$

where $d_4 = 1.0$

$$d_3 = 2 \int_2 \sqrt{K_2} (1 + \frac{1}{M_1})$$

$$d_2 = K_2 + \frac{1}{M_1} + \frac{K_2}{M_1}$$

$$d_1 = 2 \int_2 \frac{\sqrt{K_2}}{M_1}$$

$$d_0 = \frac{K_2}{M_1}$$

Now transfer function for this system.

$$\frac{X_2}{X}(\emptyset) = \frac{1}{M_1} (K_2 + 2 \int_2 \sqrt{K_2} \emptyset) / D(\emptyset)$$

$$\frac{X_1}{X}(\emptyset) = \frac{1}{M_1} (\emptyset^2 + K_2 + 2 \int_2 \sqrt{K_2} \emptyset) / D(\emptyset)$$

2.2.2 System with Absorber Attached to Sprung Mass

Equations of motion for this system as shown in Figure 3b will be:

$$m_1 \ddot{x}_1 = k_1 (x_1 - x_2) - k_2 (x_1 - x_2) - c_2 (\dot{x}_2 - \dot{x}_3) \quad \text{--- (6)}$$

$$m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) - k_3 (x_2 - x_3) - c_3 (\dot{x}_2 - \dot{x}_3) \quad \text{--- (7)}$$

$$m_3 \ddot{x}_3 = k_3 (x_2 - x_3) + c_3 (\dot{x}_2 - \dot{x}_3) \quad \text{--- (8)}$$

Applying the same non-dimensional parameters and with some simple mathematical operations

$$\ddot{x}_1 = \frac{1}{M_1} (x - x_1) - \frac{K_2}{M_1} (x_1 - x_2) - 2 \zeta_2 \frac{\sqrt{K_2}}{M_1} (\dot{x}_1 - \dot{x}_2) \quad \text{--- (9)}$$

$$\begin{aligned} \ddot{x}_2 = K_2 (x_1 - x_2) + 2 \zeta_2 \sqrt{K_2} (\dot{x}_1 - \dot{x}_2) - K_3 (x_2 - x_3) - \\ 2 \zeta_3 \sqrt{K_3} (\dot{x}_2 - \dot{x}_3) \quad \text{--- (10)} \end{aligned}$$

$$\ddot{x}_3 = \frac{K_3}{M_3} (x_2 - x_3) + 2 \zeta_3 \frac{\sqrt{K_3}}{M_3} (\dot{x}_2 - \dot{x}_3) \quad \text{--- (11)}$$

After laplace transformation and writing in matrix form

$$\begin{bmatrix} \theta^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 \zeta_2 \frac{\sqrt{K_2}}{M_1} \theta & -(\frac{K_2}{M_1} + 2 \zeta_2 \frac{\sqrt{K_2}}{M_1} \theta) & 0 \\ -(K_2 + 2 \zeta_2 \sqrt{K_2} \theta) & \theta^2 + K_2 + 2 \zeta_2 \sqrt{K_2} \theta + K_3 + 2 \zeta_3 \sqrt{K_3} \theta & \\ & -(K_3 + 2 \zeta_3 \sqrt{K_3} \theta) & \\ 0 & -(\frac{K_3}{M_3} + 2 \zeta_3 \frac{\sqrt{K_3}}{M_3} \theta) & \theta^2 + \frac{K_3}{M_3} + 2 \zeta_3 \frac{\sqrt{K_3}}{M_3} \theta \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{x}{M_1} \\ 0 \\ 0 \end{bmatrix}$$

Determinant of the matrix.

$$D(\emptyset) = d_6 \emptyset^6 + d_5 \emptyset^5 + d_4 \emptyset^4 + d_3 \emptyset^3 + d_2 \emptyset^2 + d_1 \emptyset + d_0 \quad \text{-- (12)}$$

where

$$d_6 = 1.0$$

$$d_5 = 2 \zeta_2 \sqrt{K_2} \left(1 + \frac{1}{M_1}\right) + 2 \zeta_3 \sqrt{K_3} \left(1 + \frac{1}{M_3}\right)$$

$$d_4 = \frac{K_3}{M_3} + K_2 + K_3 + \frac{1}{M_1} + \frac{K_2}{M_1} + 4 \zeta_2 \zeta_3 \sqrt{K_2 K_3} \left(\frac{1}{M_3} + \frac{1}{M_1} + \frac{1}{M_1 M_3}\right)$$

$$d_3 = 2 \zeta_3 K_2 \sqrt{K_3} \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}\right) + 2 \zeta_2 \sqrt{K_2} K_3 \left(\frac{1}{M_3} + \frac{1}{M_1 M_3} + \frac{1}{M_1}\right) + 2 \zeta_3 \sqrt{K_3} \left(\frac{1}{M_1} + \frac{1}{M_1 M_3}\right) + 2 \zeta_2 \frac{\sqrt{K_2}}{M_1}$$

$$d_2 = \frac{K_2 K_3}{M_1} + \frac{K_3}{M_1 M_3} + \frac{K_2}{M_1} + \frac{K_2 K_3}{M_1 M_3} + \frac{K_2 K_3}{M_1} + \frac{K_3}{M_1} + 4 \zeta_2 \zeta_3 \frac{\sqrt{K_2 K_3}}{M_1 M_3}$$

$$d_1 = 2 \zeta_3 \frac{K_2 \sqrt{K_3}}{M_1 M_3} + 2 \zeta_2 \frac{\sqrt{K_2} K_3}{M_1 M_3}$$

$$d_0 = \frac{K_2 K_3}{M_1 M_3}$$

Transfer function for this system

$$\frac{X_1}{X}(\emptyset) = [(\emptyset^2 + K_2 + 2 \zeta_2 \sqrt{K_2} \emptyset + K_3 + 2 \zeta_3 \sqrt{K_3} \emptyset) \left(\emptyset^2 + \frac{K_3}{M_3} + 2 \zeta_3 \frac{\sqrt{K_3}}{M_3} \emptyset\right) / M_1] / D(\emptyset) \quad \text{-- (13)}$$

$$\frac{x_2}{x}(\varnothing) = [(K_2 + 2 \int_2^{\sqrt{K_2}} \varnothing) (\varnothing^2 + \frac{K_3}{M_3} + 2 \int_3^{\sqrt{K_3}} \varnothing) / M_1] / D(\varnothing) \quad -- (14)$$

$$\frac{x_3}{x}(\varnothing) = [(K_2 + 2 \int_2^{\sqrt{K_2}} \varnothing) (\frac{K_3}{M_3} + 2 \int_3^{\sqrt{K_3}} \varnothing) / M_1] / D(\varnothing) \quad -- (15)$$

2.2.3 System with Absorber Attached to Unsprung Mass

Equations of motion for the system as shown in Figure 3c are

$$m_1 \ddot{x}_1 = k_1 (x - x_1) - k_2 (x_1 - x_2) - c_2 (\dot{x}_1 - \dot{x}_2) - c_3 (\dot{x}_1 - \dot{x}_3) - k_3 (x_1 - x_3) \text{ ---- (16)}$$

$$m_2 \ddot{x}_2 = k_2 (x_1 - x_2) + c_2 (\dot{x}_1 - \dot{x}_2) \text{ ---- (17)}$$

$$m_3 \ddot{x}_3 = k_3 (x_1 - x_3) + c_3 (\dot{x}_1 - \dot{x}_3) \text{ ---- (18)}$$

Applying the previously mentioned non-dimensional parameters and simple mathematical operations.

$$\begin{aligned} \ddot{x}_1 &= \frac{1}{M_1} (x - x_1) - \frac{K_2}{M_1} (x_1 - x_2) - 2 \zeta_2 \frac{\sqrt{K_2}}{M_1} (\dot{x}_1 - \dot{x}_2) \\ &\quad - \frac{K_3}{M_3} (x_1 - x_3) - 2 \zeta_3 \frac{\sqrt{K_3}}{M_1} (\dot{x}_1 - \dot{x}_3) \\ \ddot{x}_2 &= K_2 (x_1 - x_2) + 2 \zeta_2 \sqrt{K_2} (\dot{x}_1 - \dot{x}_2) \\ \ddot{x}_3 &= \frac{K_3}{M_3} (x_1 - x_3) + 2 \zeta_3 \frac{\sqrt{K_3}}{M_3} (\dot{x}_1 - \dot{x}_3) \text{ ---- (19)} \end{aligned}$$

After the laplace transformation and writing the equations in the matrix form.

$$\begin{bmatrix} \emptyset^2 + \frac{1}{M_1} + \frac{K_2}{M_1} + 2 \delta_2 \sqrt{\frac{K_2}{M_1}} \emptyset + \frac{K_3}{M_1} + 2 \delta_3 \sqrt{\frac{K_3}{M_1}} \emptyset - \left(\frac{K_2}{M_1} + 2 \delta_2 \sqrt{\frac{K_2}{M_1}} \emptyset \right) - \left(\frac{K_3}{M_1} + 2 \delta_3 \sqrt{\frac{K_3}{M_1}} \emptyset \right) \\ - (K_2 + 2 \delta_2 \sqrt{K_2} \emptyset) \quad \emptyset^2 + K_2 + 2 \delta_2 \sqrt{K_2} \emptyset \quad 0 \\ - \left(\frac{K_3}{M_3} + 2 \delta_3 \sqrt{\frac{K_3}{M_3}} \emptyset \right) \quad 0 \quad \emptyset^2 + \frac{K_3}{M_3} + 2 \delta_3 \sqrt{\frac{K_3}{M_3}} \emptyset \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{x}{M_1} \\ 0 \\ 0 \end{bmatrix}$$

Determinant of the matrix

$$D(\emptyset) = d_6 \emptyset^6 + d_5 \emptyset^5 + d_4 \emptyset^4 + d_3 \emptyset^3 + d_2 \emptyset^2 + d_1 \emptyset + d_0 \quad \text{----- (20)}$$

where

$$d_6 = 1.0$$

$$d_5 = 2 \delta_2 \sqrt{K_2} \left(1 + \frac{1}{M_1} \right) + 2 \delta_3 \sqrt{K_3} \left(\frac{1}{M_1} + \frac{1}{M_3} \right)$$

$$d_4 = \frac{1}{M_1} + K_2 \left(\frac{1}{M_1} + 1 \right) + K_3 \left(\frac{1}{M_1} + \frac{1}{M_3} \right) + 4 \delta_2 \delta_3 \sqrt{K_2} \sqrt{K_3} \left(\frac{1}{M_1 M_3} + \frac{1}{M_1} + \frac{1}{M_3} \right)$$

$$d_3 = 2 \oint_2 \sqrt{K_2} \left(\frac{1}{M_1} + K_3 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) \right) + 2 \oint_3 \sqrt{K_3} \left(\frac{1}{M_1 M_3} + K_2 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) \right)$$

$$d_2 = 4 \oint_2 \oint_3 \frac{\sqrt{K_2} \sqrt{K_3}}{M_1 M_3} + K_2 K_3 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right) + \frac{K_3}{M_1 M_3} + \frac{K_2}{M_1}$$

$$d_1 = (2 \oint_2 \sqrt{K_2} K_3 + 2 \oint_3 \sqrt{K_3} K_2) / M_1 M_3$$

$$d_0 = K_2 K_3 / M_1 M_3$$

Main transfer functions for this system are

$$\frac{X_1(\emptyset)}{X} = \frac{(\emptyset^2 + K_2 + 2 \oint_2 \sqrt{K_2} \emptyset) (\emptyset^2 + \frac{K_3}{M_3} + 2 \oint_3 \frac{\sqrt{K_3}}{M_3} \emptyset) / M_1}{D(\emptyset)}$$

$$\frac{X_2(\emptyset)}{X} = \frac{(K_2 + 2 \oint_2 \sqrt{K_2} \emptyset) (\emptyset^2 + \frac{K_3}{M_3} + 2 \oint_3 \frac{\sqrt{K_3}}{M_3} \emptyset) / M_1}{D(\emptyset)}$$

$$\frac{X_3(\emptyset)}{X} = \frac{\emptyset^2 + K_2 + 2 \oint_2 \sqrt{K_2} \emptyset) (\frac{K_3}{M_3} + 2 \oint_3 \frac{\sqrt{K_3}}{M_3} \emptyset) / M_1}{D(\emptyset)} \quad (21)$$

2.3 Vehicle Disturbances

In actual environments vehicle suspension systems are subjected to multiple input disturbances. However, in this study, emphasis is placed on guideway disturbances resulting from terrain irregularity. This guideway disturbance is treated as random and is described by its power spectral density.

Experimental data shows that for wide variety of surfaces the spectra may be well approximated by a hyperbolic displacement density function.

$$S(\Omega) = \frac{A}{\Omega^2}$$

where "A" is roughness parameter in m. and " Ω " is spatial frequency rad./m.

However on applying this form of spectra, it should be kept in mind that there will be some situations in which (A/Ω^2) would not fit the real roadway spectrum very well. Also for very long or very short wave length (A/Ω^2) may represent extrapolation of data, which may not very well be justified. Finally any one dimensional spectrum represents only an incomplete description of roadway, even in the statistical sense. However the use of such form has the great advantage that all roadways smooth

or rough are represented by a single parameter A , thus rather general preliminary design studies may be made which should have wide applicability.

If the vehicle traverses the surface with a constant forward velocity V , and w is the circular frequency in time, the height of the roadway under the vehicle may be described by random process in time, realizing that

$$V \Omega = w \quad \text{-----} (22)$$

The spectral density in spatial domain may be converted to a spectral density in the time domain.

$$\text{as } S_x(\Omega) d\Omega = S_x(w) dw \quad \text{-----} (23)$$

$$V d\Omega = dw$$

$$\frac{A}{\Omega^2} d\Omega = S_x(w) dw$$

$$S_x(w) = \frac{A}{V \Omega^2} = \frac{A}{V \left(\frac{w}{V}\right)^2}$$

$$\text{hence } S_x(w) = \frac{AV}{w^2}$$

in non dimensional form

$$S_x(\gamma) = \frac{S_x(w)}{S_x(w_{nn})} = \frac{1}{\gamma^2}$$

where $\gamma = \frac{w}{w_{nn}}$

and $S_x(w_{nn}) = \frac{AV}{w_{nn}^2}$

by using expression similar to eq. (22), this gives

$$x_0 = \sqrt{AV/w_{nn}}$$

for a hyperbolic displacement spectral density input.

2.4 Behavior Variable Representation

Since the input to the vehicle is considered to be random excitation, the behavior variable of the response are expected to be random as well. Thus the response can only be described in terms of any of the following statistical parameters:

- i) RMS value.
- ii) Amplitude probability distribution, expressed as probability density.
- iii) Vibration spectrum, continuous in frequency, expressed as spectral density.
- iv) Autocorrelation function.

RMS magnitude may be considered as the most convenient statistical parameter that can be selected for the behavior variable representation. Since it is the only statistical quantitative parameter and also because, in general, all other statistical parameters can be expressed in terms of RMS.

From the autocorrelation definition

$$R(\tau) = E[x(t) \cdot x(t+\tau)] \quad \text{----- (28)}$$

If the time of delay τ is brought to zero, then

$$R(0) = x^2(t) = \int S_x(\omega) d\omega = \sigma^2 \quad \text{----- (29)}$$

where $R(\tau)$ is the Autocorrelation function and σ^2 is the standard deviation which can define any function with probability distribution considered to be one of the mathematical known function (Gaussian or Rayleigh).

However, when the behavior variable is strongly frequency dependent, we therefore need to know more about the frequency distribution, i.e., it would be advisable to study the power spectral density together with the RMS value as representation of behavior variable.

Since the behavior variable can be expressed in terms of the integral of the square magnitude of the transfer function between the behavior variable and the excitation multiplied by the spectral density of the disturbance input, as will be shown later. Thus behavior variable could be obtained in the form of total square integral. Consequently the magnitude of the RMS value of any of the behavior variables could be evaluated through the direct application of the method developed by Booten and Mathew (38).

In general, this method obtains the value of the complex line integral.

$$I_n = \frac{1}{2\pi j} \int_{-\infty j}^{\infty j} \frac{a(s) \cdot a(-s)}{d(s) \cdot d(-s)} ds \quad \text{----- (30A)}$$

where

$$a(s) = a_{n-1} s^{n-1} + a_{n2} s^{n2} + \dots + a_0$$

$$d(s) = d_n s^n + d_{n-1} s^{n-1} + \dots + d_0 \quad \text{----- (30B)}$$

A table which lists the values of these integrals for several values of n can be found in the same ref. (22).

2.5 Vehicle Performance

The vehicle performance and vibration characteristics are obtained based on wheel controllability (ride safety) and ride comfort.

2.5.1 Wheel Controllability and Ride Safety

Since the dynamic variation of normal force between the tire of the wheel and the terrain (ground force), in general determines tire-terrain contact area during normal operation, and since traction characteristics which involve side thrust and self aligning torque are dependent on that contact area; directional control and skid resistance of a wheel, in turn, is governed by the traction characteristics. It follows that vehicle controllability can be indicated by the tire-terrain normal force.

Also, since the increase of the variation of the tire-terrain dynamic force relative to static force would increase the change in wheel load, and consequently the possibility of the wheel to leave the ground; the ground force variation of the dynamic tire-terrain normal force with respect to the static wheel force could be

considered a good indicator of ride safety.

2.5.2 Ride Comfort

This is one of the basic goals to be provided by vehicle suspension system, particularly in the case of passenger vehicles. The use of integral square of vehicle body acceleration in case of deterministic input, such as isolated bumps and obstacles, has the advantage of giving greater weight to the large values of acceleration resulting from initial impact than to the smaller acceleration experienced during the subsequent decay of the oscillation. In addition, it has been shown that force transmission to the human body is, in the case of vertical vibration an appropriate measure of discomfort. At frequency up to 5 Hz the force transmission is about the same as if the human body were replaced by pure mass. In this range, therefore the acceleration of the passenger may be taken as proportional to the force transmission, and hence as a measure of discomfort. Thus RMS sprung mass acceleration may be considered as a good criterion of the ride comfort.

However, human vibration sensitivity depends not only on the magnitude of vibratory acceleration, but also on its frequency. Consequently, the power spectral density of the vehicle acceleration would have the same weight as the RMS value.

2.6 Evaluation of Objective Function

2.6.1 Objective Function

The objective function is formed of RMS ground force (as measure of controllability and ride safety), and is constrained by vehicle acceleration (as a measure of ride comfort). Thus the optimization problem will have the form:

Find minimum F_g, x, rms

Subjected to constraint

$$G = \ddot{X}_2, x, \text{rms} - \ddot{Y}_{C',x',\text{rms}} \leq 0$$

where F_g, x, rms and $\ddot{X}_2, x, \text{rms}$ are the non-dimensional RMS ground force and the RMS unsprung mass acceleration respectively, and $\ddot{Y}_{C',x',\text{rms}}$ is the non-dimension RMS comfort limit defined by

$$\ddot{Y}_{C',x',\text{rms}} = \frac{\ddot{Y}_C}{\sigma \sqrt{AV w_{nn}}^3}$$

where \ddot{Y}_C is the maximum allowable acceleration level in dimensional term, and σ is the multiple of standard deviation required to ensure a reasonable probability of not exceeding the constraint.

2.6.2 Evaluation

For the application of parameter search optimization techniques the performance variables formulating

the objective function (ground force and body acceleration) should be defined in terms of vehicle design parameters.

The RMS of the non-dimensional ground force can be obtained by

$$\bar{F}_g, x, \text{rms} = \left[\int_{-\infty}^{\infty} |H_{fg,x}(\gamma)|^2 S_x(\gamma) d\gamma \right]^{1/2} \quad \text{----- (31)}$$

where $H_{fg,x}(\gamma)$ is the non-dimensional transfer function between the ground force and input excitation, and $S_x(\gamma)$ is the non-dimensional displacement spectral density.

From eq. (24) and replacing γ by \emptyset/j we get

$$\bar{F}_g, x, \text{rms} = \left[\frac{1}{j} \int_{-\infty j}^{\infty j} |H_{fg,x}(\emptyset) H_{fg,x}(-\emptyset)| \frac{1}{-\emptyset^2} d\emptyset \right]^{1/2} \quad \text{-- (32)}$$

$$\text{Noting that } fg = K_1 (X - X_1) \quad \text{---- (33)}$$

$$\text{and } F_g = \frac{f_g}{K_1 X_0} \quad \text{---- (34)}$$

$$\text{So } F_g = X - X_1 \quad \text{---- (35)}$$

$$\text{and } H_{fg,x}(\emptyset) = 1 - H_{x_1 x}(\emptyset) \quad \text{---- (36)}$$

where $H_{x_1 x}(\emptyset)$ is defined already by eq. (13)

Consequently, the RMS value is evaluated by applying Bootons method (22)

$$F_{g'x'rms} = \sqrt{2\pi I_n} \quad \text{----- (37)}$$

In the case of conventional system (without absorber):

$$I_n = I_4$$

$$I_4 = a_3^2 (-d_0^2 d_3 + d_0 d_1 d_2) + (a_2^2 - 2a_1 a_3) d_0 d_1 d_4 + (a_1^2 - 2a_0 a_2) d_0 d_3 d_4 \quad \text{-- (38)}$$

$$+ a_0^2 (d_1 d_4^2 + d_2 d_3 d_4) / 2d_0 d_4 (-d_0 d_3^2 - d_1^2 d_4 + d_1 d_2 d_3)$$

where

$$a_3 = 1.0$$

$$a_2 = 2 \int_0^{\infty} \sqrt{k_2} \left(1 + \frac{1}{M_1}\right)$$

$$a_1 = K_2 \left(1 + \frac{1}{M_1}\right)$$

$$a_0 = 0$$

and d's are defined by equation (5).

Similarly the RMS value of sprung mass acceleration can be obtained as

$$\ddot{x}_{2'x'rms} = \left[\int_{-\infty}^{\infty} H_{\ddot{x}_{2,x}}(\gamma)^2 S_x(\gamma) d\gamma \right]^{1/2} \quad \text{----- (40)}$$

where $H_{\ddot{x}_2', x}(\gamma)$ is the non-dimensional transfer function between the sprung mass acceleration and input excitation. Following the same procedure, we get

$$\ddot{x}_{2', x', \text{rms}} = \sqrt{2\pi I_n} \quad \text{----- (41)}$$

where I_n is defined by eq. (38)

with

$$a_3 = 0.0$$

$$a_2 = 2 \int_0^{\infty} \sqrt{K_2/M_1} \, d\gamma$$

$$a_1 = K_2/M_1$$

$$a_0 = 0$$

and d 's are defined by eq. (5)

Thus selecting a suitable value of $\ddot{y}_{c', x', \text{rms}}$ the objective function will be known and expressed in terms of design parameters.

In the case where absorber is attached to sprung mass:

$$I_n = I_6$$

$$F_{g'x'rms} = \sqrt{2\pi I_6}$$

where

$$I_6 = \frac{1}{2P_6} [a_5^2 q_0 + (a_4^2 - 2a_3a_5)q_1 + (a_3^2 - 2a_2a_4 + 2a_1a_5)q_2 \\ + (a_2^2 - 2a_1a_3 + 2a_0a_4)q_3 + (a_1^2 - 2a_0a_2)q_4 + a_0^2 q_5] \quad \text{---- (42)}$$

where

$$q_0 = \frac{1}{d_6} (d_4q_1 - d_2q_2 + d_0q_3)$$

$$q_1 = -d_0d_1d_5 + d_0d_3^2 - d_1d_2d_3 + d_1^2d_4$$

$$q_2 = d_0d_3d_5 + d_1^2d_6 - d_1d_2d_5$$

$$q_3 = d_0d_5^2 + d_1d_3d_6 - d_1d_4d_5$$

$$q_4 = \frac{1}{d_0} (d_2q_3 - d_4q_2 + d_6q_1)$$

$$q_5 = \frac{1}{d_0} (d_2q_4 - d_4q_3 + d_6q_2)$$

$$p_6 = d_0 (d_1q_5 - d_3q_4 + d_5q_3)$$

with

$$a_5 = 1.0$$

$$a_4 = 2 \oint_3 \sqrt{K_3} \left(1 + \frac{1}{M_3}\right) + 2 \oint_2 \sqrt{K_2} \left(1 + \frac{1}{M_1}\right)$$

$$a_3 = 4 \oint_2 \oint_3 \sqrt{K_2 K_3} \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}\right) + K_3 \left(1 + \frac{1}{M_3}\right) + K_2 \left(1 + \frac{1}{M_1}\right)$$

$$a_2 = 2 \oint_2 \sqrt{K_2} K_3 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}\right) + 2 \oint_3 \sqrt{K_3} K_2 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}\right)$$

$$a_1 = K_2 K_3 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3}\right)$$

$$a_0 = 0.0$$

all d's are defined in eq. (12)

In the similar way as for the case (without absorber)

sprung mass acceleration can be obtained

$$\ddot{X}_{2',x',rms} = \sqrt{2\pi I_n}$$

now a's will be as follows

$$a_5 = 0$$

$$a_4 = 2 \oint_2 \sqrt{K_2}/M_1$$

$$a_3 = 4 \oint_2 \oint_3 \sqrt{K_2 K_3}/M_1 M_3 + K_2/M_1$$

$$a_2 = (2 f_2 K_3 \sqrt{K_2} + 2 f_3 K_2 \sqrt{K_3}) / M_1 M_3$$

$$a_1 = K_2 K_3 / M_1 M_3$$

$$a_0 = 0$$

Finally in the case where the absorber is attached to the unprung mass:

$$I_n = I_6$$

$$F_{g'x'rms} = \sqrt{2\pi I_6}$$

$$I_6 = \frac{1}{2P_6} [a_5^2 q_0 + (a_4^2 - 2a_3 a_5) q_1 + (a_3^2 - 2a_2 a_4 + 2a_1 a_5) q_2 \\ + (a_2^2 - 2a_1 a_3 + 2a_0 a_4) q_3 + (a_1^2 - 2a_0 a_2) q_4 + a_0^2 q_5]$$

where all d's are defined by eq. (20)

now for the case of ground force a's will be as follows.

$$a_5 = 1.0$$

$$a_4 = 2 f_2 \sqrt{K_2} (1 + \frac{1}{M_1}) + 2 f_3 \sqrt{K_3} (\frac{1}{M_3} + \frac{1}{M_1})$$

$$a_3 = K_2 + \frac{K_3}{M_3} + \frac{1}{M_1} (K_2 + K_3) + 4 f_2 f_3 \sqrt{K_2 K_3} (\frac{1}{M_1 M_3} + \frac{1}{M_1} + \frac{1}{M_3})$$

$$a_2 = (2 \zeta_2 \sqrt{K_2} K_3 + 2 \zeta_3 \sqrt{K_3} K_2) \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right)$$

$$a_1 = K_2 K_3 \left(\frac{1}{M_1} + \frac{1}{M_3} + \frac{1}{M_1 M_3} \right)$$

$$a_0 = 0.0$$

and for the case of unsprung mass acceleration

$$a_5 = 0$$

$$a_4 = 2 \zeta_2 K_2 / M_1$$

$$a_3 = 4 \zeta_2 \zeta_3 \frac{K_2 K_3}{M_1 M_3} + \frac{K_2}{M_1}$$

$$a_2 = (2 \zeta_2 K_3 \sqrt{K_2} + 2 \zeta_3 K_2 \sqrt{K_3}) / M_1 M_3$$

$$a_1 = K_2 K_3 / M_1 M_3$$

$$a_0 = 0$$

3. OPTIMIZATION TECHNIQUE

With the general availability of digital computers, iterative searching techniques have been developed in recent years for automatically locating the immediate neighborhood of design optimization problem solution point. Many algorithms have been developed for this purpose P519RE (2) is used in this study.

Program P519RE is derived to solve constrained and unconstrained minimization problems. The objective function is accomplished by the introduction of a penalty function "P" to account for constraint violation in the search process, greater the constraint violation the greater will be the positive value of "P".

Two basic types of techniques in search process are direct search or pattern search, and descent search or gradient based search. Direct search techniques have the advantage of avoiding gradient evaluations and of requiring little storage space in a digital computer. However, compared with gradient based search techniques, direct search techniques are generally relatively inefficient, requiring a large number of function evaluations. Also in direct search techniques, premature termination is more likely to occur.

In order to use P519RE, the specific design

optimization is reformulated to a standard format, which is then embedded in a general program for solution by a digital computer. In this study, the initial formulation of the objective function is

$$\downarrow F_{g',x',rms} = \sqrt{2 \pi AI_6}$$

where AI_6 is defined by (42). This objective function is constrained by the body acceleration;

$$\ddot{x}_{2',x',rms} = \sqrt{2 \pi BI_6}$$

where BI_6 is defined by (42). Since no further simplification is required in this initial formulation, it will also be the final formulation, i.e.

$$Q \downarrow = \downarrow F_{g',x',rms} = \sqrt{2 \pi AI_6}$$

where AI_6 is expressed in terms of unitized variables given in Appendix II.

3.1 With Absorber Attached to the System

Given Constants:

$$CG(1) = M_1 = 0.1$$

$$CG(2) = M_3 = 0.01$$

$$CG(3) = K_{2max} = 1.0$$

$$CG(4) = K_{2\min} = 0.001$$

$$CG(5) = f_{2\max} = 1.0$$

$$CG(6) = f_{2\min} = 0.001$$

$$CG(7) = K_{3\max} = 1.0$$

$$CG(8) = K_{3\min} = 0.001$$

$$CG(9) = f_{3\max} = 1.0$$

$$CG(10) = f_{3\min} = 0.001$$

$$CG(11) = \ddot{y}_{c'x'rms} = 0.3$$

Constraints:

$$K_{2\min} \leq K_2 \leq K_{2\max}$$

$$K_{3\min} \leq K_3 \leq K_{3\max}$$

$$f_{2\min} \leq f_2 \leq f_{2\max}$$

$$f_{3\min} \leq f_3 \leq f_{3\max}$$

$$\ddot{x}_{2'x'rms} \leq \ddot{y}_{c'x'rms}$$

Variables:

$$K_2, K_3, f_2, f_3$$

Unitized Variables:

$$V(1) = \frac{K_2}{K_{2\max}}$$

$$V(2) = \frac{K_3}{K_{3\max}}$$

$$V(3) = \frac{f_2}{f_{2\max}}$$

$$V(4) = \frac{f_3}{f_{3\max}}$$

Calculated Constants:

$$C(1) = \frac{K_{2\max}}{K_{2\min}} = CG(3)/CG(4)$$

$$C(2) = \frac{K_{3\max}}{K_{3\min}} = CG(5)/CG(6)$$

$$C(3) = \frac{f_{2\max}}{f_{2\min}} = CG(7)/CG(8)$$

$$C(4) = \frac{f_{3\max}}{f_{3\min}} = CG(9)/CG(10)$$

Regional Constraints:

$$R(1) = K_{2\max} - K_2 \geq 0$$

$$R(2) = K_2 - K_{2\min} \geq 0$$

$$R(3) = K_{3\max} - K_3 \geq 0$$

$$R(4) = K_3 - K_{3\min} \geq 0$$

$$R(5) = f_{2\max} - f_2 \geq 0$$

$$R(6) = f_2 - f_{2\min} \geq 0$$

$$R(7) = f_{3\max} - f_3 \geq 0$$

$$R(8) = f_3 - f_{3\min} \geq 0$$

$$R(9) = \ddot{y}_{c'x'rms} - \ddot{x}_{2'x'rms} \geq 0$$

Unitized Constraints:

$$1 - V(1) \geq 0$$

$$V(1) \times C(1) - 1 \geq 0$$

$$1 - V(2) \geq 0$$

$$V(2) \times C(2) - 1 \geq 0$$

$$1 - V(3) \geq 0$$

$$V(3) \times C(3) - 1 \geq 0$$

$$1 - V(4) \geq 0$$

$$V(4) \times C(4) - 1 \geq 0$$

$$1 - ACC/CG(11) \geq 0$$

Final Output:

$$F(1) = Q = \text{Objective (ground force)}$$

$$F(2) = K_2 = V(1) \times CG(3)$$

$$F(3) = K_3 = V(2) \times CG(5)$$

$$F(4) = f_2 = V(3) \times CG(7)$$

$$F(5) = f_3 = V(4) \times CG(9)$$

$$F(6) = M_1 = CG(1)$$

$$F(7) = M_3 = CG(2)$$

$$F(8) = \ddot{y}_{c'x'rms} = CG(11)$$

3.2 Without Absorber

Given Constants:

$$CG(1) = M_1 = 0.1$$

$$CG(2) = K_{2min} = 0.001$$

$$CG(3) = K_{2max} = 1.0$$

$$CG(4) = f_{2min} = 0.001$$

$$CG(5) = f_{2max} = 1.0$$

$$CG(6) = \ddot{y}_{c'x'rms} = 0.3$$

Constraints:

$$K_{2max} \geq K_2 \geq K_{2min}$$

$$f_{2max} \geq f_2 \geq f_{2min}$$

$$\ddot{x}_{2'x'rms} \leq \ddot{y}_{c'x'rms}$$

Variables:

$$K_2, f_2$$

Unitized Variables:

$$V(1) = \frac{K_2}{K_{2max}}$$

$$V(2) = \frac{f_{2x'}}{f_{2max}}$$

Calculated Constants:

$$C(1) = \frac{K_{2\max}}{K_{2\min}} = CG(2)/CG(3)$$

$$C(2) = \frac{f_{2\max}}{f_{2\min}} = CG(4)/CG(5)$$

Regional Constraints:

$$R(1) = K_{2\max} - K_2 \geq 0$$

$$R(2) = K_2 - K_{2\min} \geq 0$$

$$R(3) = f_{2\max} - f_2 \geq 0$$

$$R(4) = f_2 - f_{2\min} \geq 0$$

$$R(5) = \ddot{y}_{c'x'rms} - \ddot{x}_{2'x'rms} \geq 0$$

Unitized Constraints:

$$1 - V(1) \geq 0$$

$$V(1) \times C(1) - 1 \geq 0$$

$$1 - V(2) \geq 0$$

$$V(2) \times C(2) - 1 \geq 0$$

$$1 - ACC/CG(6) \geq 0$$

Final Output:

$$F(1) = Q = \text{Objective (ground force)}$$

$$F(2) = K_2 = V(1) \times CG(2)$$

$$F(3) = f_2 = V(2) \times CG(4)$$

$$F(4) = M_1 = CG(1)$$

$$F(5) = \ddot{y}_{c'x'rms} = CG(6)$$

RESULTS & DISCUSSIONS

4.1 Optimum Behavior

4.1.1 Trade-Off Curves

The optimum ground force sprung mass acceleration trade-off curves for both optimum damped absorber suspension systems, for different values of M_1 and M_3 including $M_3=0$ (the conventional system), are shown in Figures 4-12. Corresponding values of the ground forces and sprung mass accelerations are given in Tables 1-9.

From these figures, it is clear that wheel controllability at a specific ride comfort level improves with decreasing M_1 for both damped absorber systems. Also, wheel controllability improves with increasing M_3 , which imposes a compromise upon choosing M_3 : improving controllability without adding much weight to the vehicle.

In addition, Figures 4-12 demonstrate that the optimum damped absorber systems behave better than the conventional one, i.e., adding damped absorber improves controllability at a given ride comfort. The figures also demonstrate that the optimum damped absorber with the absorber attached to the unsprung mass is superior to that with the absorber attached to the sprung mass.

TABLE - 1 -

$M_1 = 0.25$ $M_3 = 0.01$			
$\ddot{Y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	5.23749	4.97892	3.00328
0.4	3.95264	3.80632	2.73087
0.5	3.19136	3.10742	2.49529
0.6	2.70292	2.64907	2.29889
0.7	2.37668	2.33969	2.14131
0.8	2.15765	2.13252	2.02195
0.9	2.01463	1.99820	1.93841
1.0	1.92807	1.91828	1.88755
1.1	1.88459	1.88008	1.86543

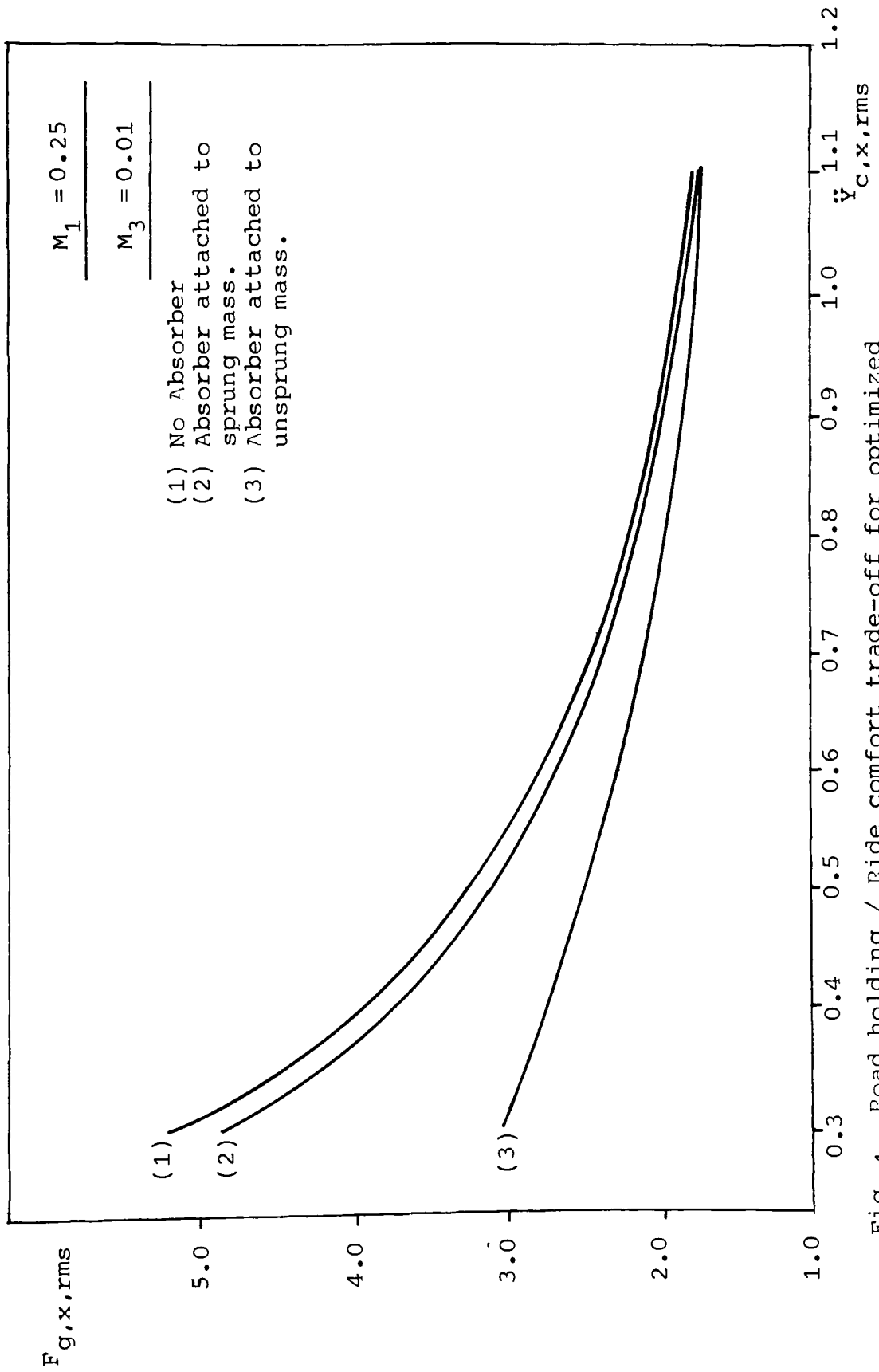


Fig. 4 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 2 -

$M_1 = 0.25$ $M_3 = 0.025$			
$\ddot{Y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	5.23749	4.70892	2.59597
0.4	3.95264	3.65868	2.42013
0.5	3.19136	3.00751	2.26561
0.6	2.70292	2.58099	2.13491
0.7	2.37668	2.29359	2.02910
0.8	2.15765	2.10111	1.94897
0.9	2.01463	1.97742	1.89423
1.0	1.92807	1.90603	1.86326
1.1	1.88459	1.87487	1.85549

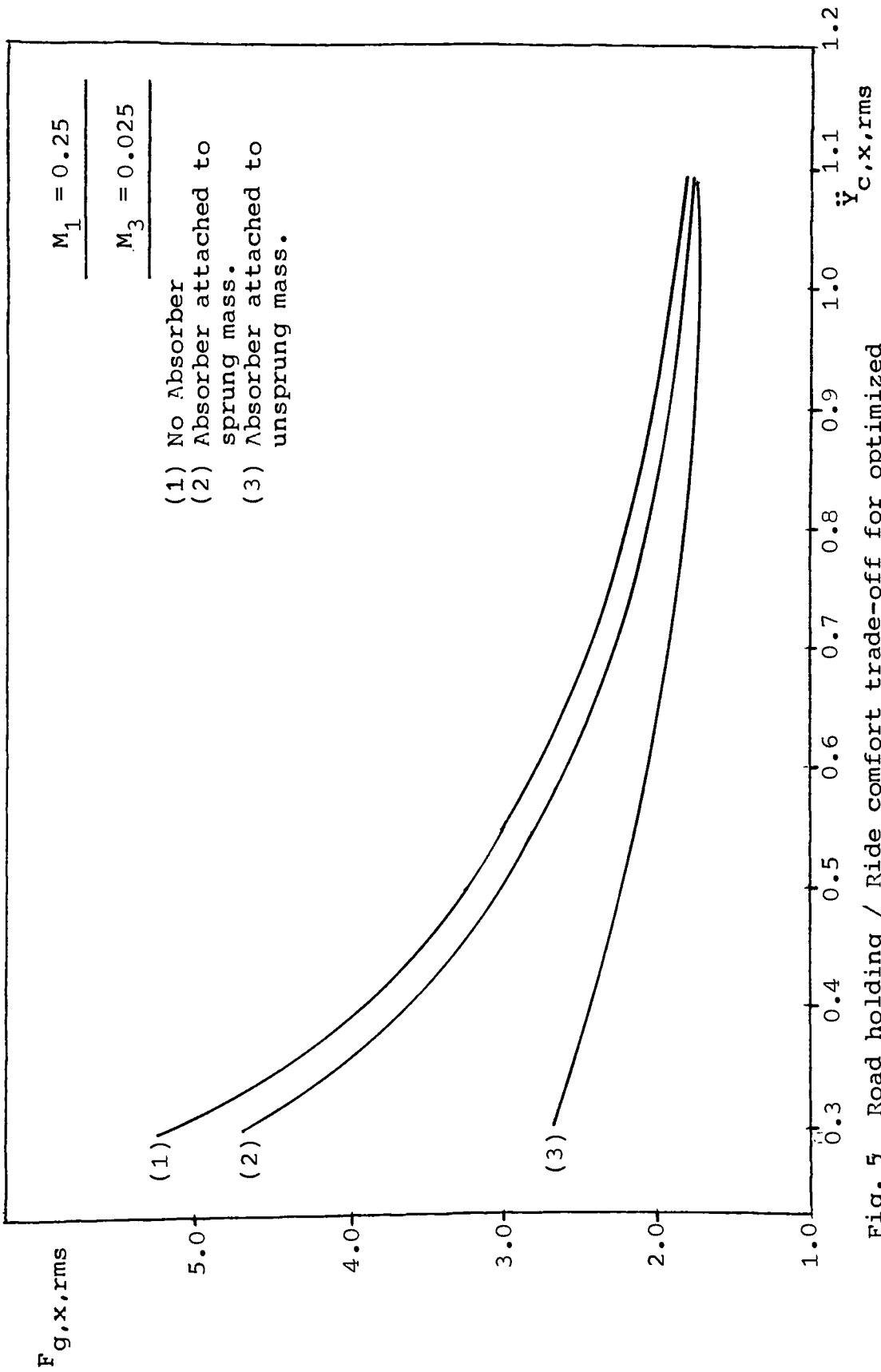


Fig. 5 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 3 -

$M_1 = 0.25$ $M_3 = 0.05$			
$\ddot{y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	5.23749	4.43130	2.33592
0.4	3.95264	3.47709	2.21379
0.5	3.19136	2.88454	2.10709
0.6	2.70292	2.49393	2.01747
0.7	2.37668	2.23134	1.94620
0.8	2.15765	2.05768	1.89418
0.9	2.01463	1.94902	1.86160
1.0	1.92807	1.89003	1.84805
1.1	1.88459	1.87113	1.84752

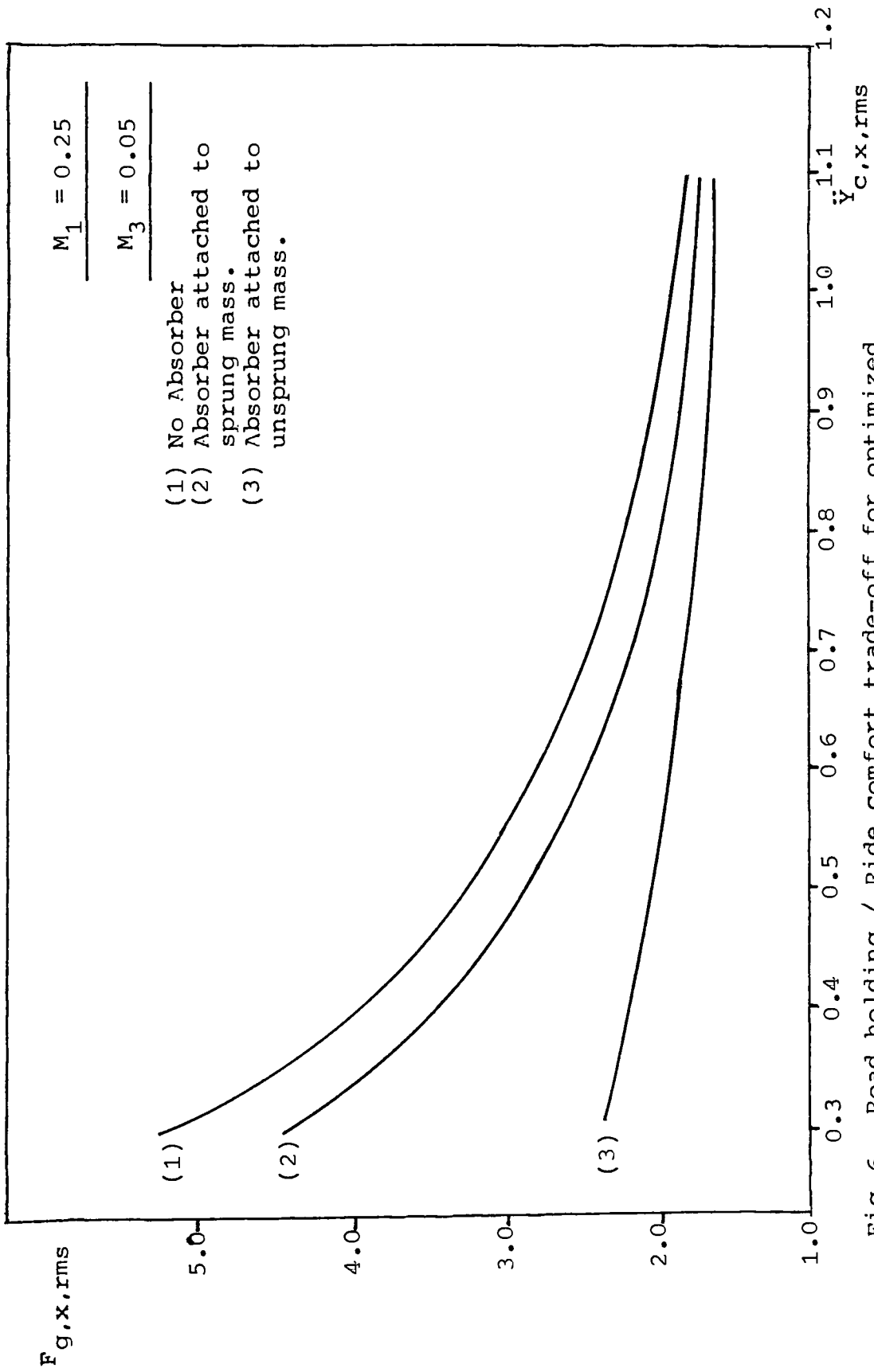


Fig.6 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 4 -

$M_1 = 0.1$ $M_3 = 0.005$			
$\ddot{y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	3.32713	3.17697	2.15168
0.4	2.51880	2.48751	1.93231
0.5	2.05509	2.03621	1.75400
0.6	1.77150	1.75852	1.61808
0.7	1.59791	1.59021	1.52296
0.8	1.49872	1.49447	1.46530
0.9	1.45240	1.45091	1.44039
1.0	1.44357	1.44213	1.43834
1.1	1.44357	1.44200	1.43565

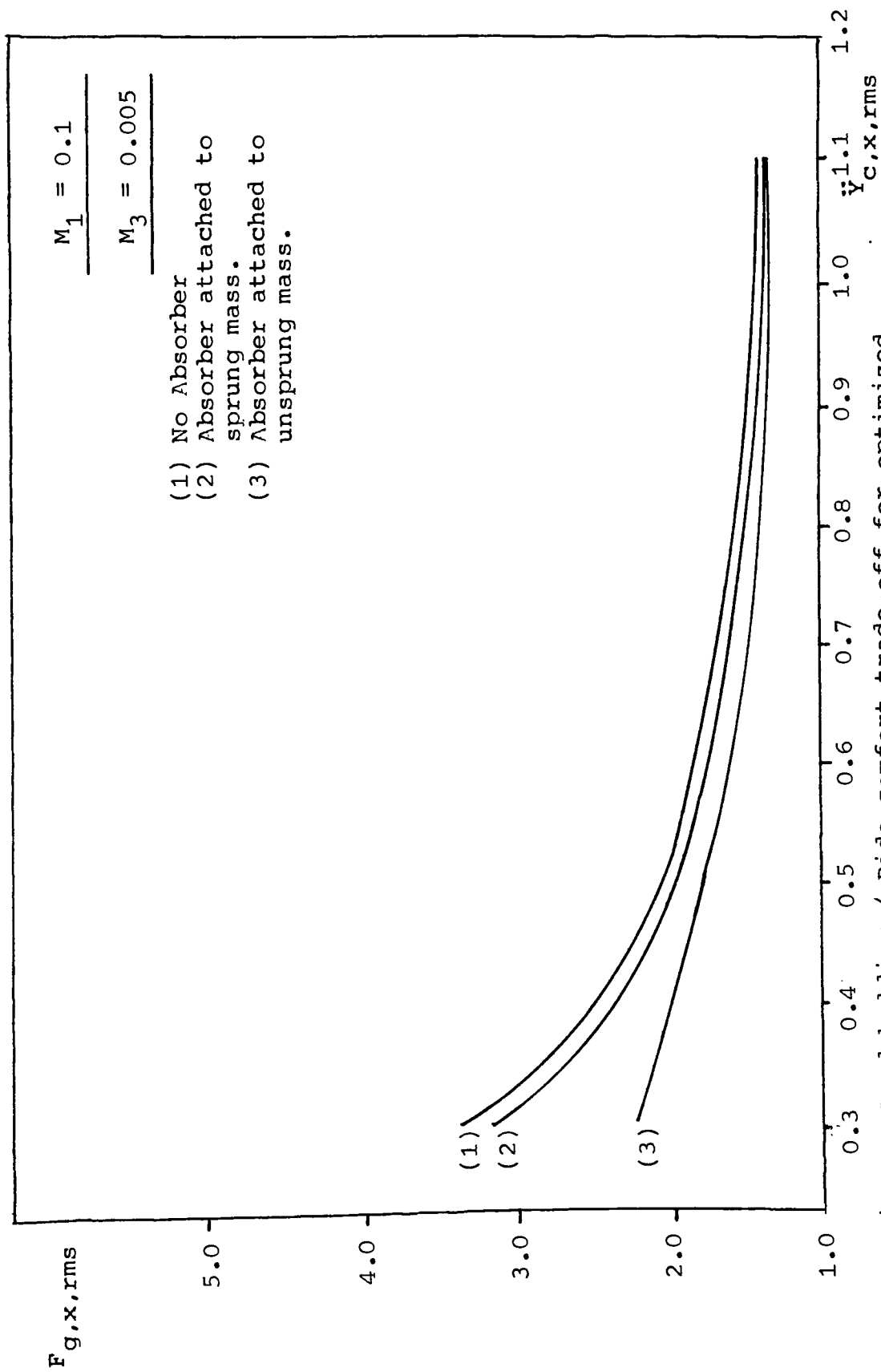


Fig. 7.7 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 5 -

$M_1 = 0.1$ $M_3 = 0.01$			
$\ddot{Y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	3.32713	3.29605	1.95528
0.4	2.51880	2.49523	1.79597
0.5	2.05509	2.02359	1.66518
0.6	1.77150	1.74958	1.56420
0.7	1.59791	1.58801	1.49342
0.8	1.49872	1.49056	1.45156
0.9	1.45240	1.44999	1.43596
1.0	1.44357	1.44320	1.43565
1.1	1.44357	1.44319	1.43565

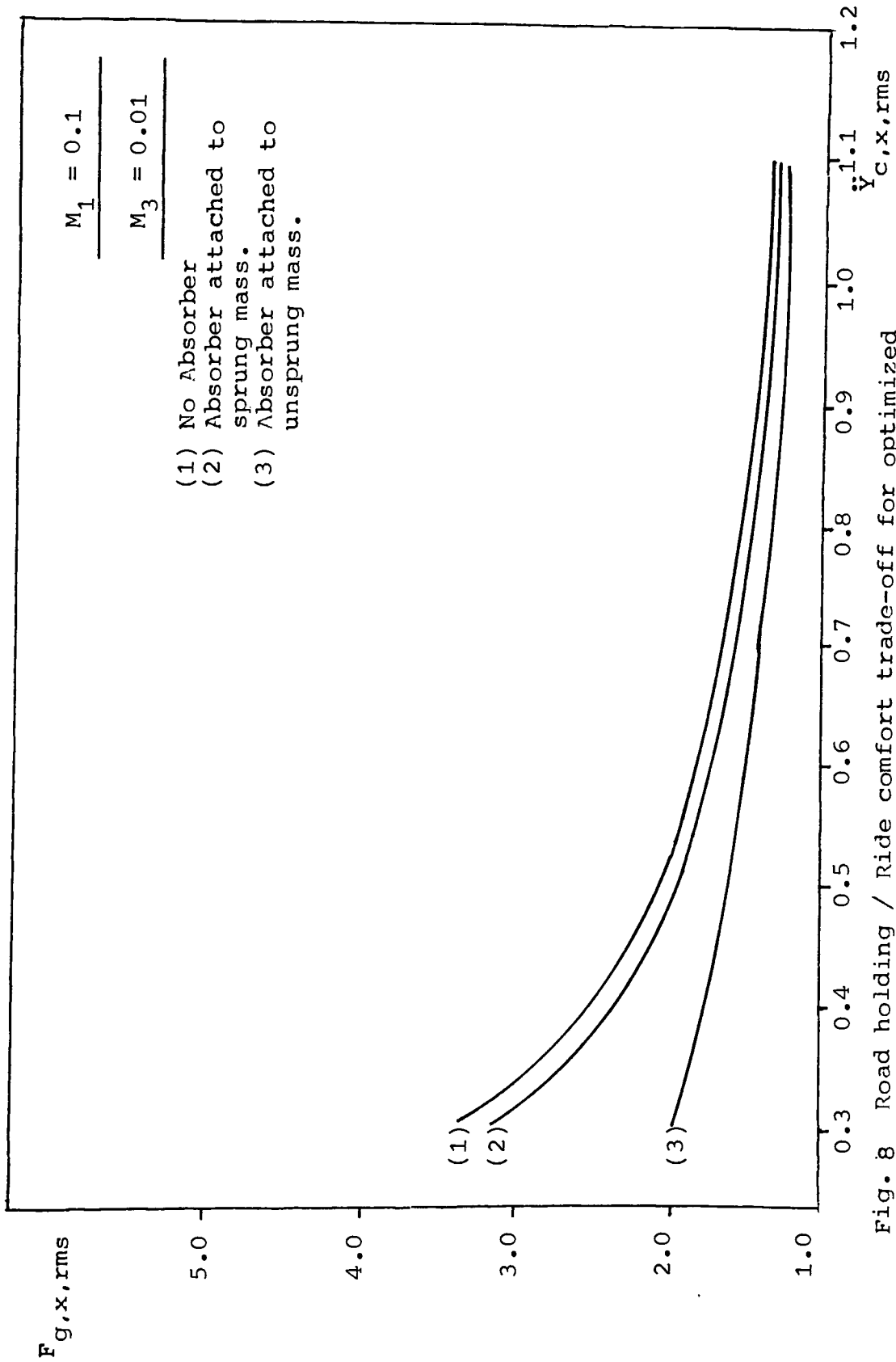


Fig. 8 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 6 -

$M_1 = 0.1$ $M_3 = 0.025$			
$\ddot{Y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	3.32713	3.22979	1.73289
0.4	2.51880	2.46118	1.63587
0.5	2.05509	2.01146	1.55649
0.6	1.77150	1.73878	1.49624
0.7	1.59791	1.57546	1.45590
0.8	1.49872	1.48506	1.43514
0.9	1.45240	1.44697	1.43149
1.0	1.44357	1.44277	1.43149
1.1	1.44357	1.44277	1.43149

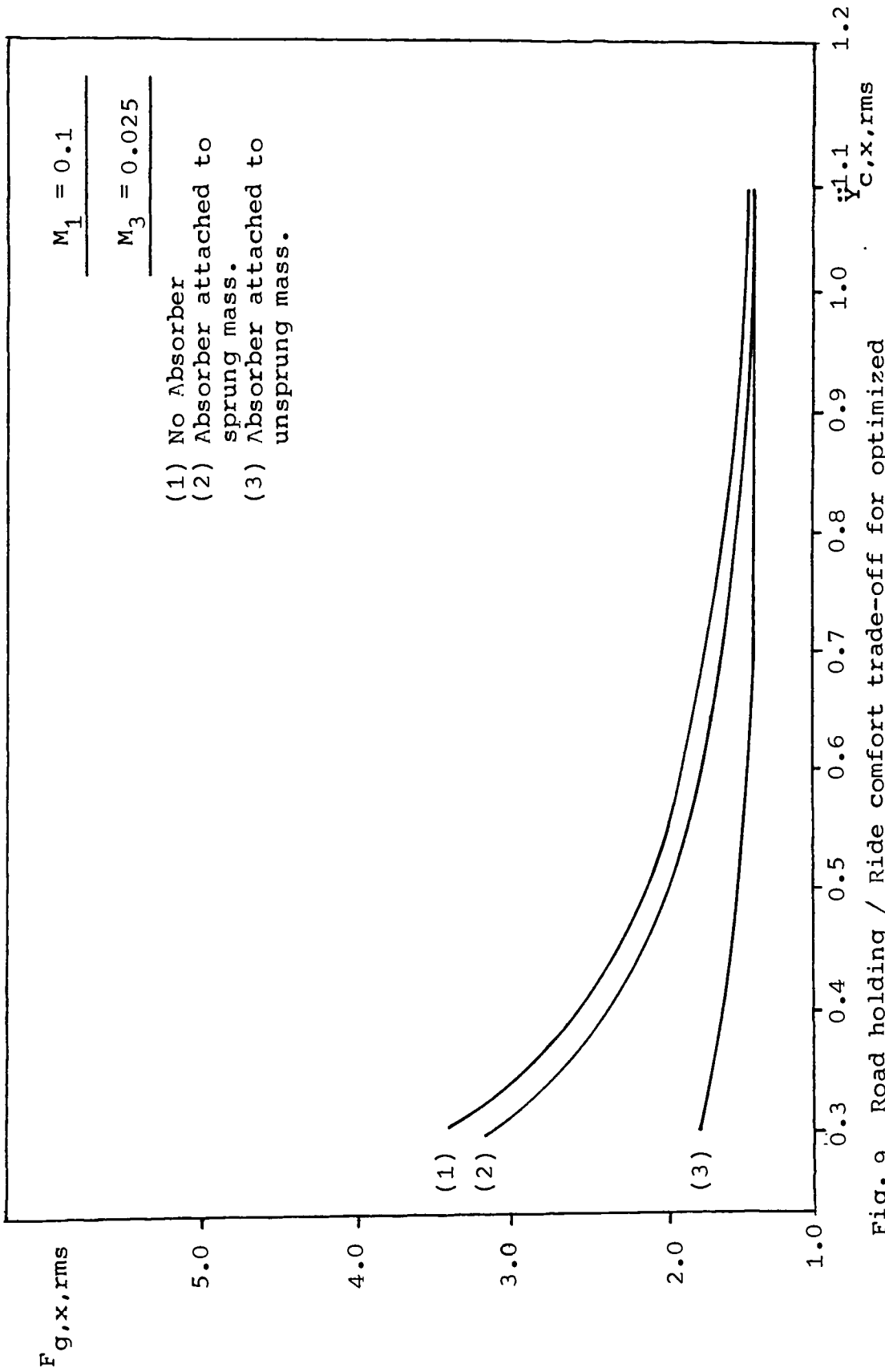


Fig. 9 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 7 -

$M_1 = 0.05$ $M_3 = 0.005$			
$\ddot{Y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	2.17312	2.14418	1.69976
0.4	1.80376	1.79930	1.50672
0.5	1.49570	1.49212	1.36257
0.6	1.32252	1.31364	1.26632
0.7	1.23356	1.23243	1.21334
0.8	1.20123	1.20124	1.19671
0.9	1.19986	1.19982	1.19669
1.0	1.19986	1.19982	1.19669
1.1	1.19986	1.19982	1.19669

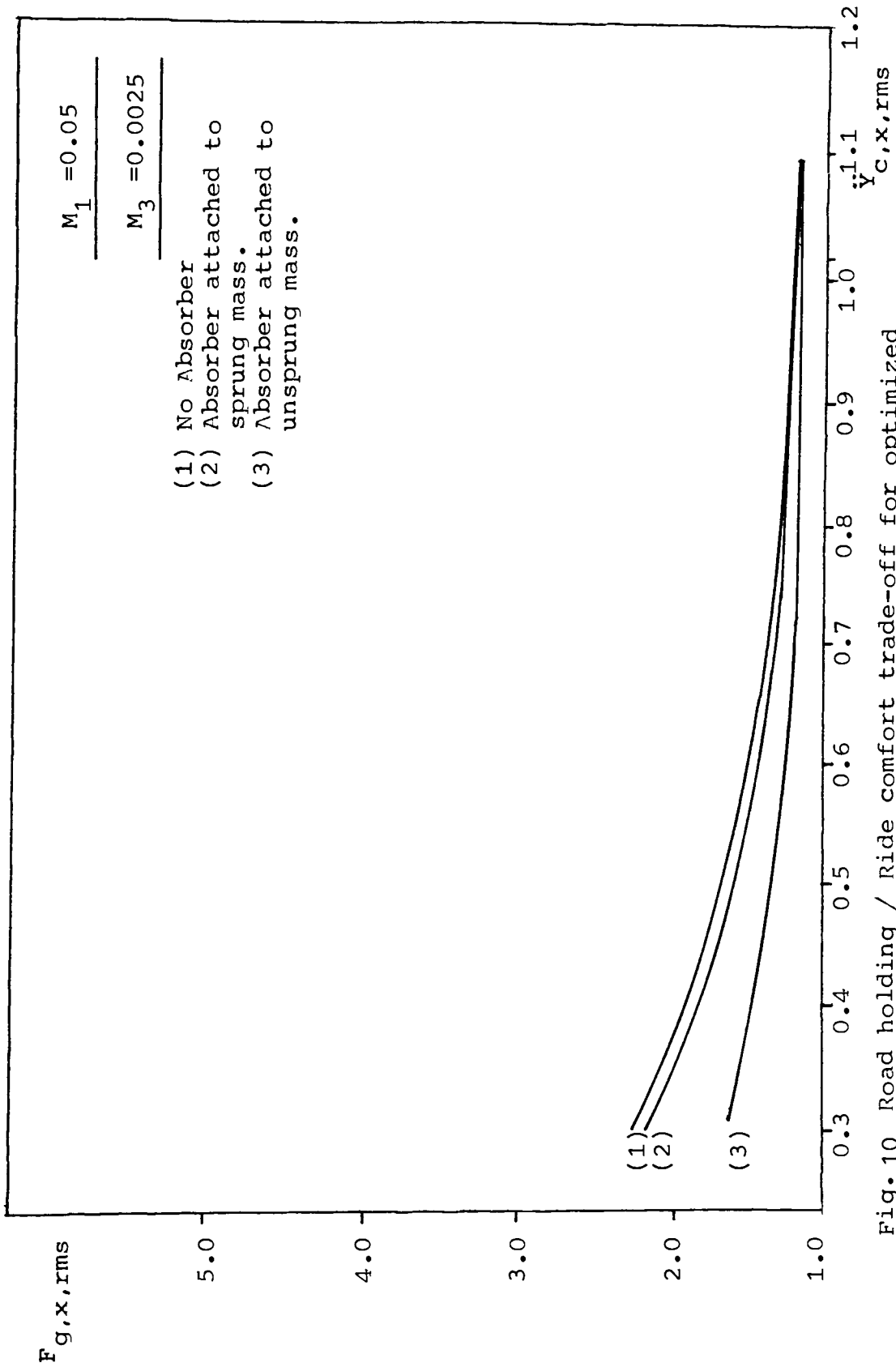


Fig. 10 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 8 -

$M_1 = 0.05$ $M_3 = 0.005$			
$\ddot{Y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	2.17312	2.02343	1.56498
0.4	1.80376	1.79572	1.42350
0.5	1.49570	1.48973	1.31621
0.6	1.32252	1.31866	1.24398
0.7	1.23356	1.23152	1.20507
0.8	1.20128	1.20083	1.19514
0.9	1.19986	1.19775	1.19514
1.0	1.19986	1.19735	1.19514
1.1	1.19986	1.19735	1.19514

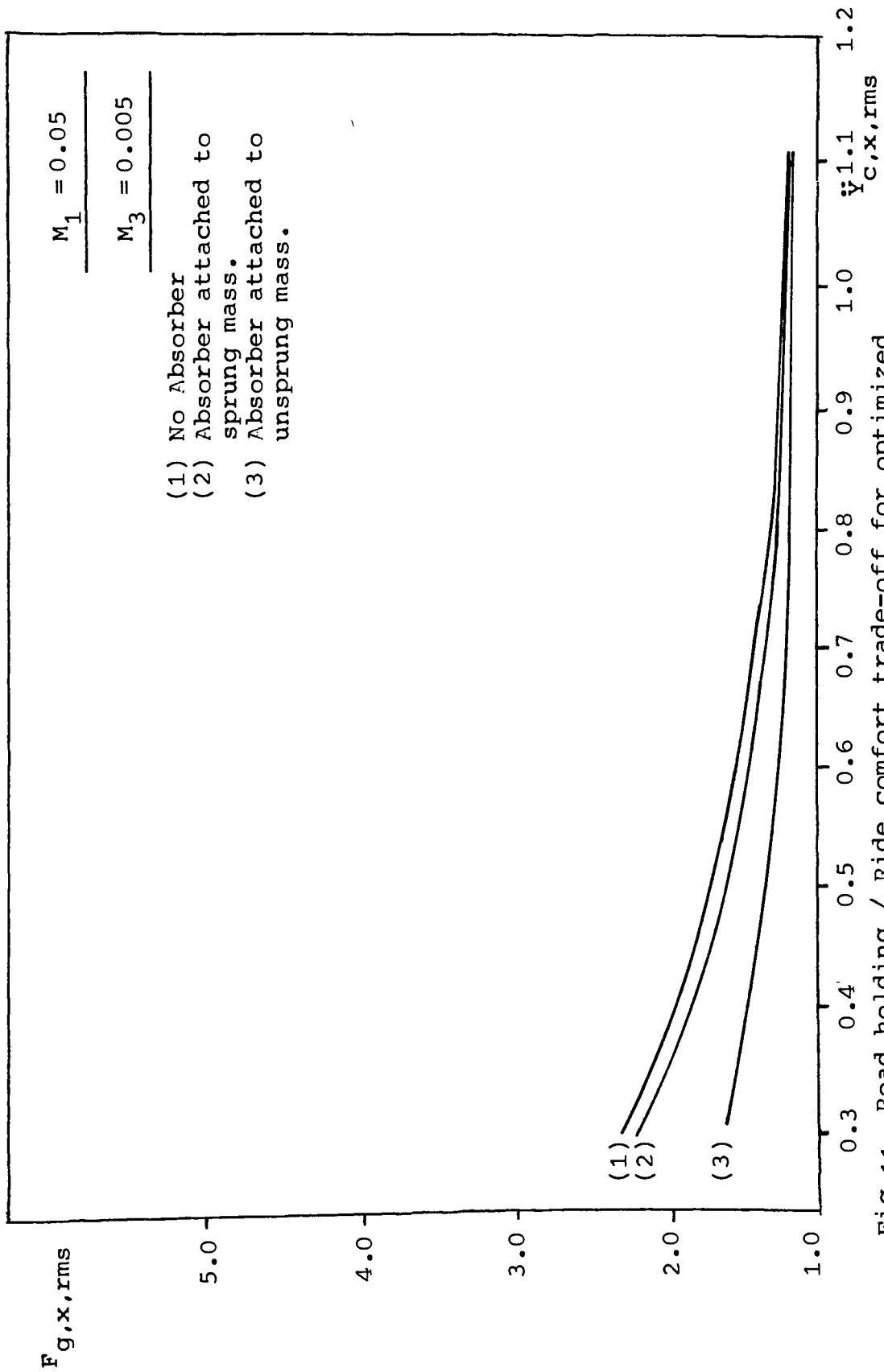


Fig.11 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

TABLE - 9 -

$M_1 = 0.05$ $M_3 = 0.01$			
$\ddot{y}_{c,x,rms}$	$F_{g,x,rms}$		
	Without absorber	With absorber attached to	
		Sprung mass	Unsprung mass
0.3	2.17312	2.014164	1.44396
0.4	1.80376	1.78966	1.34547
0.5	1.49570	1.48408	1.27115
0.6	1.32252	1.31501	1.22199
0.7	1.23356	1.22957	1.19745
0.8	1.20128	1.20044	1.19344
0.9	1.19986	1.20044	1.19344
1.0	1.19986	1.20044	1.19344
1.1	1.19986	1.20044	1.19344

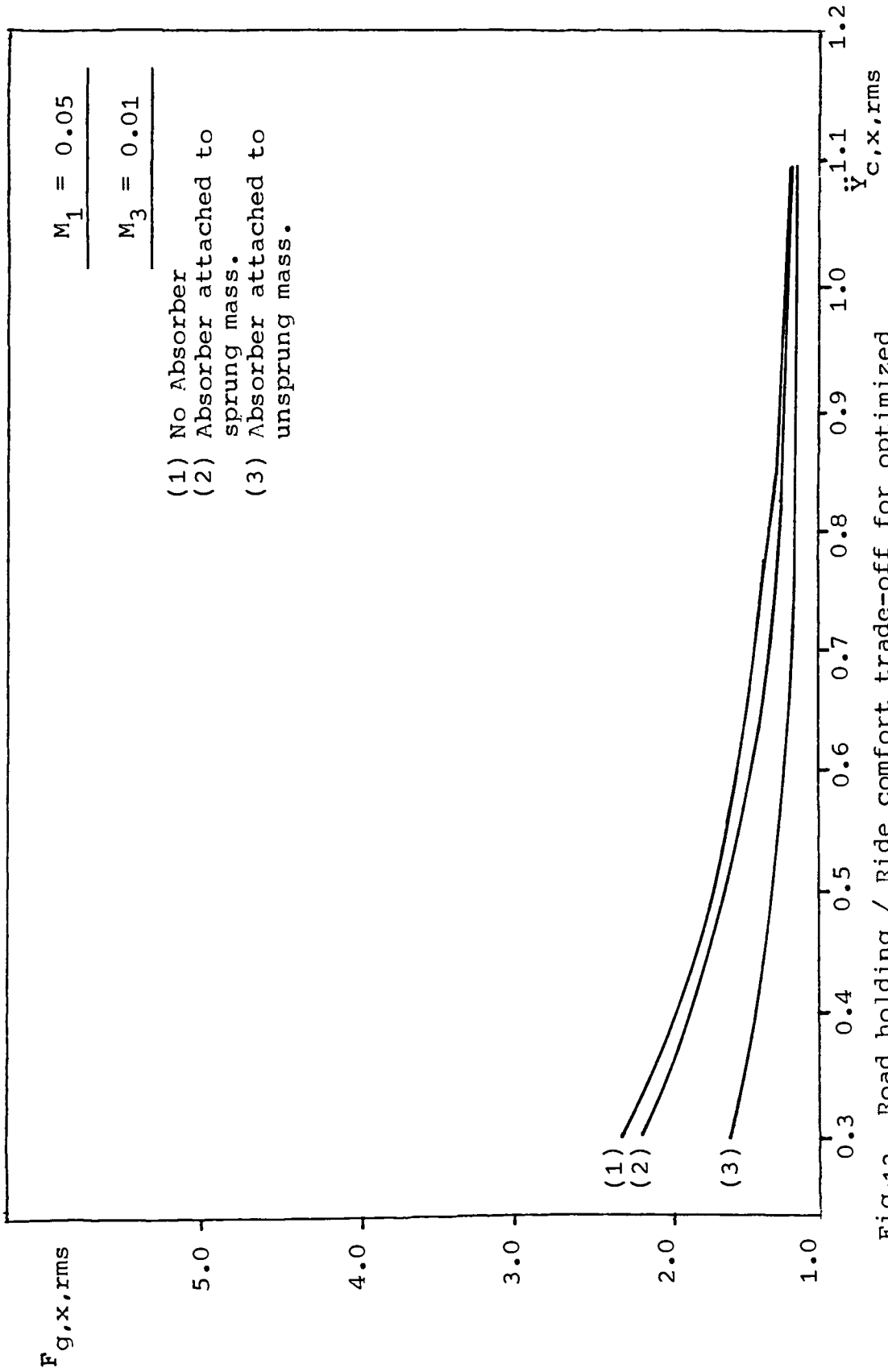


Fig.12 Road holding / Ride comfort trade-off for optimized conventional and optimized damped suspension systems.

4.1.2 Power Spectral Density

Figures 13-18 display the PSD of the sprung mass acceleration at the constrained optimum case, for specified values of M_1 and for $\mu=0.01$ ($\mu=\frac{M_1}{M_3}$) which is the most practical ratio in applications (14, 24), for the three optimum systems: 1) the conventional, 2) the damped absorber with the absorber attached to the sprung mass, 3) with the absorber attached to the unsprung mass.

From these figures it is observed that when the optimum damped absorber is attached to the unsprung mass, the peak at the secondary resonance is suppressed. On the other hand, when the absorber is attached to the sprung mass, the peak is slightly reduced. In other words, when the absorber is attached to the unsprung mass, it is better tuned and consequently gives better ride comfort at the neighborhood of the secondary resonance.

Also, it can be shown that decreasing M_1 , in general, tends to flatten the PSD and distribute it over a wider range of frequency, i.e., decreasing M_1 for a given constrained comfort, reduces the peaks of vibration but introduces a detrimental effect on higher frequency vibration.

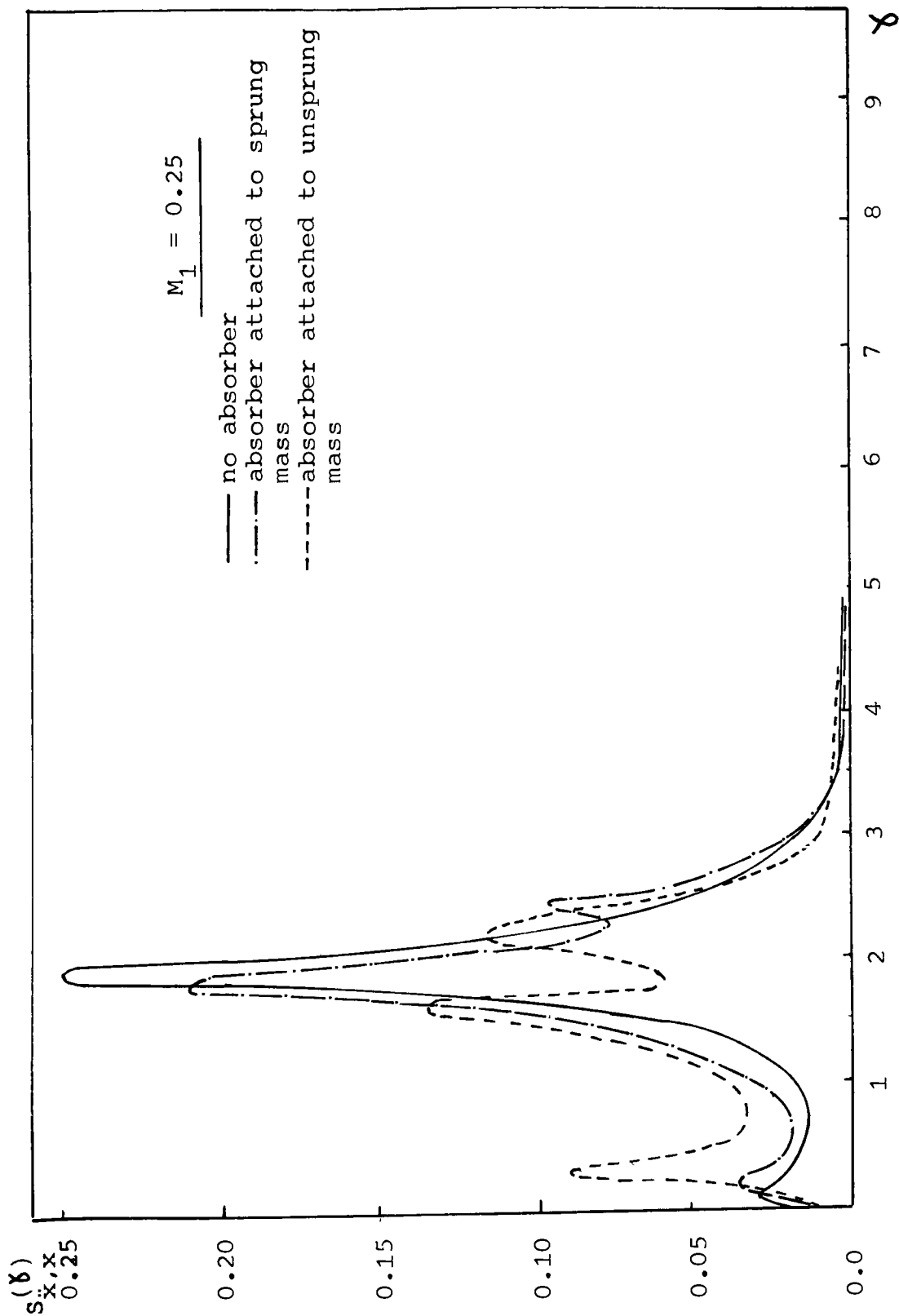


Fig. 13. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,rms} = 0.6$

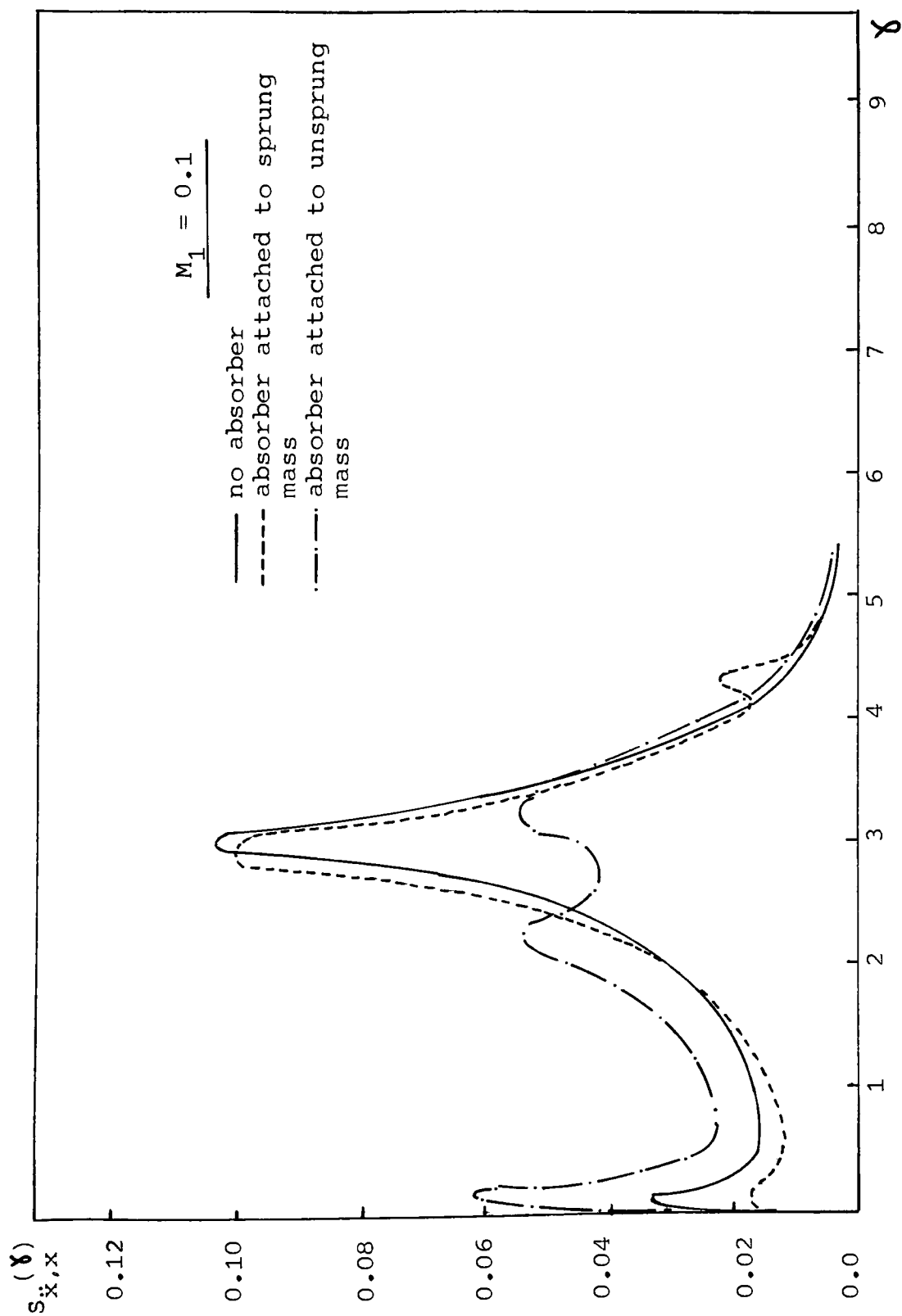


Fig. 14. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,rms} = 0.6$

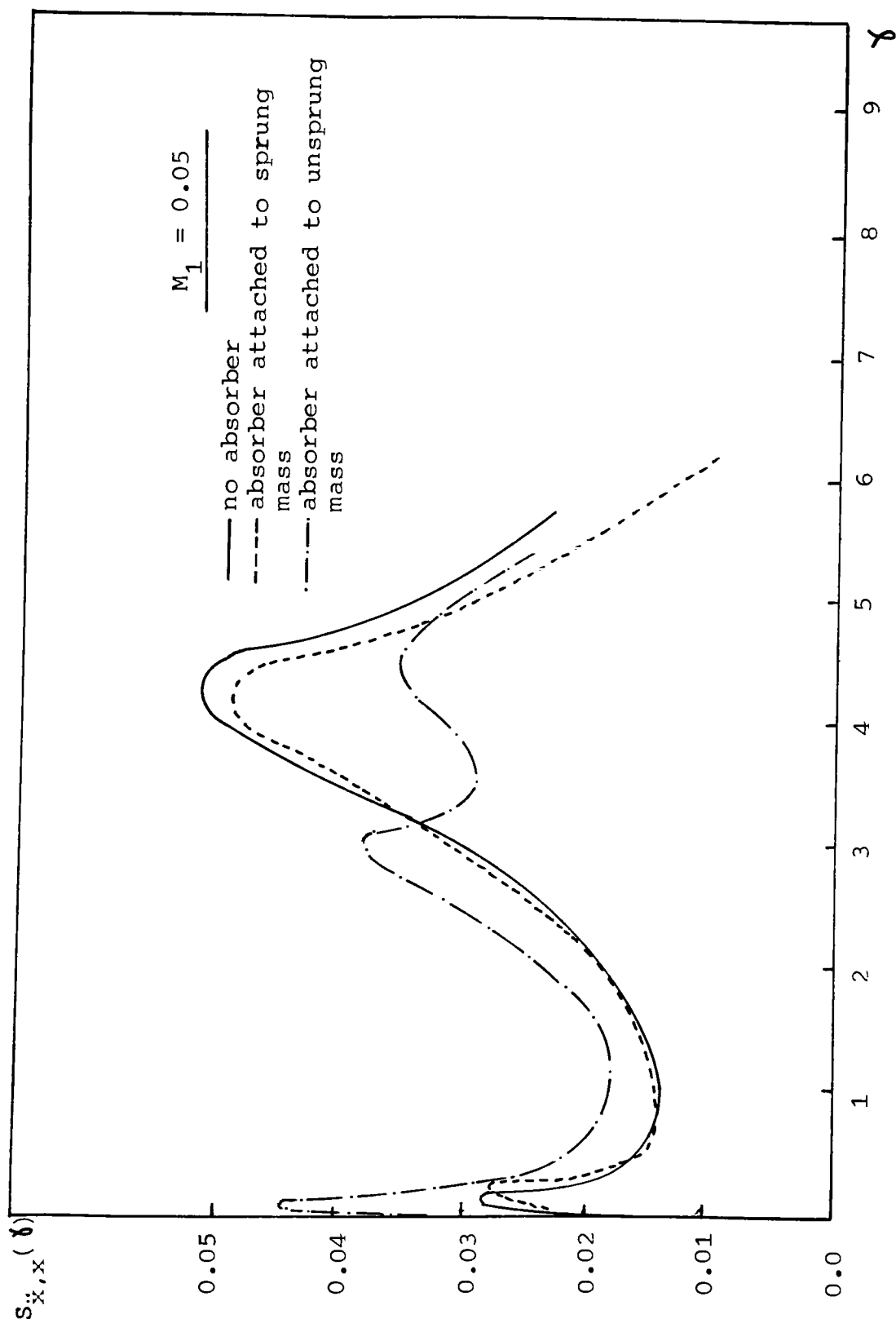


Fig.15. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,rms} = 0.6$

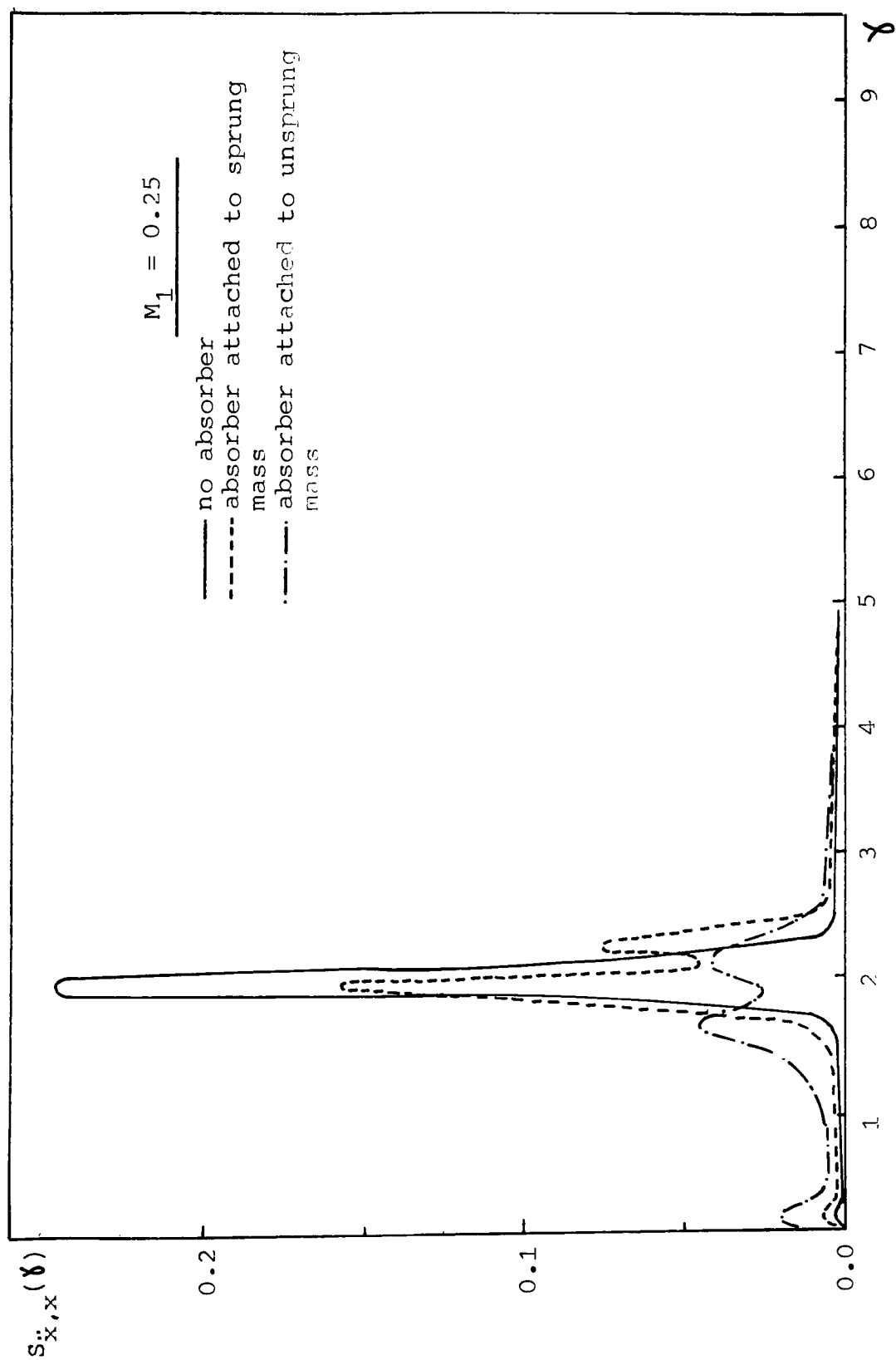


Fig.16. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,rms} = 0.3$

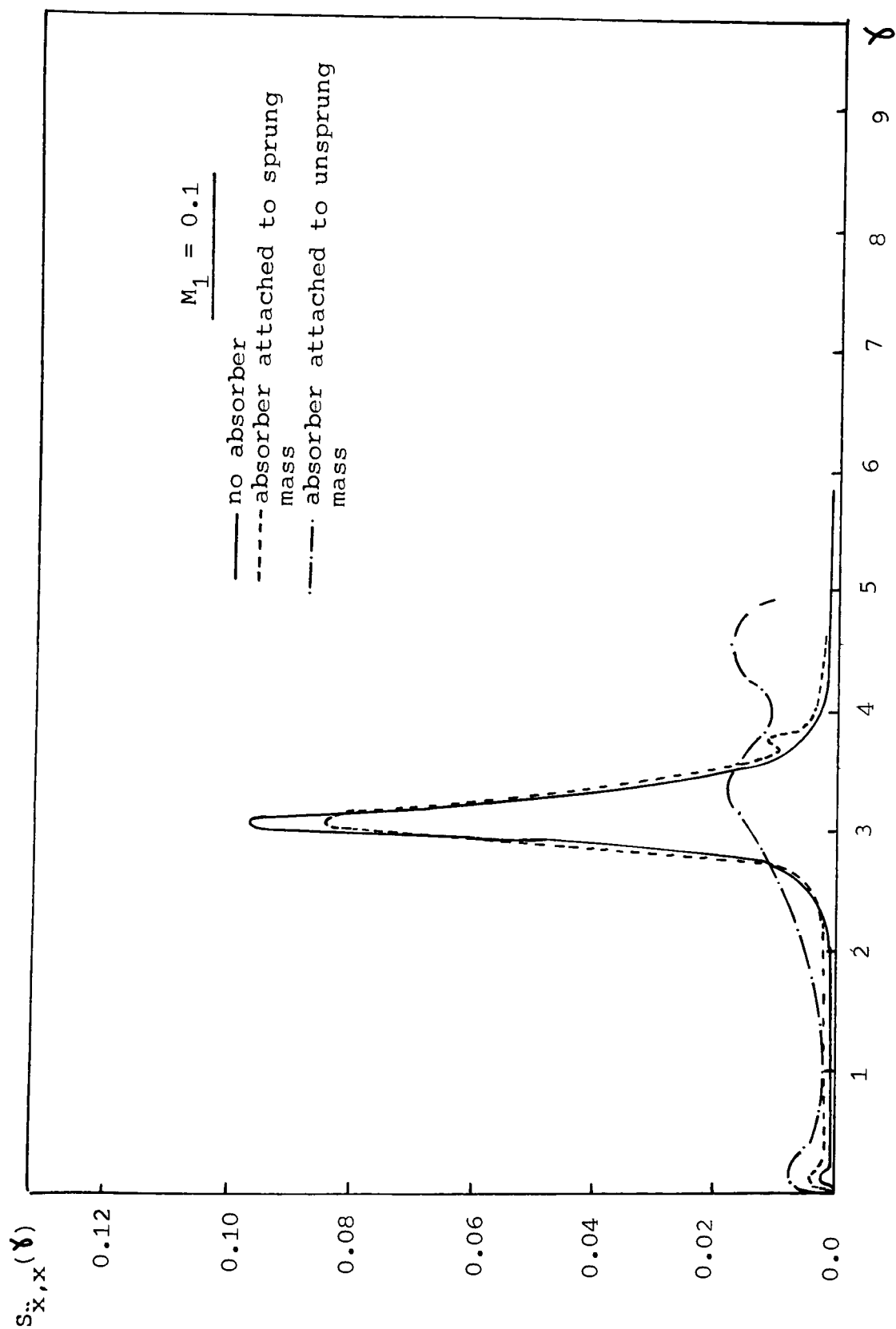


Fig. 17. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\dot{y}_{c,x,rms} = 0.3$

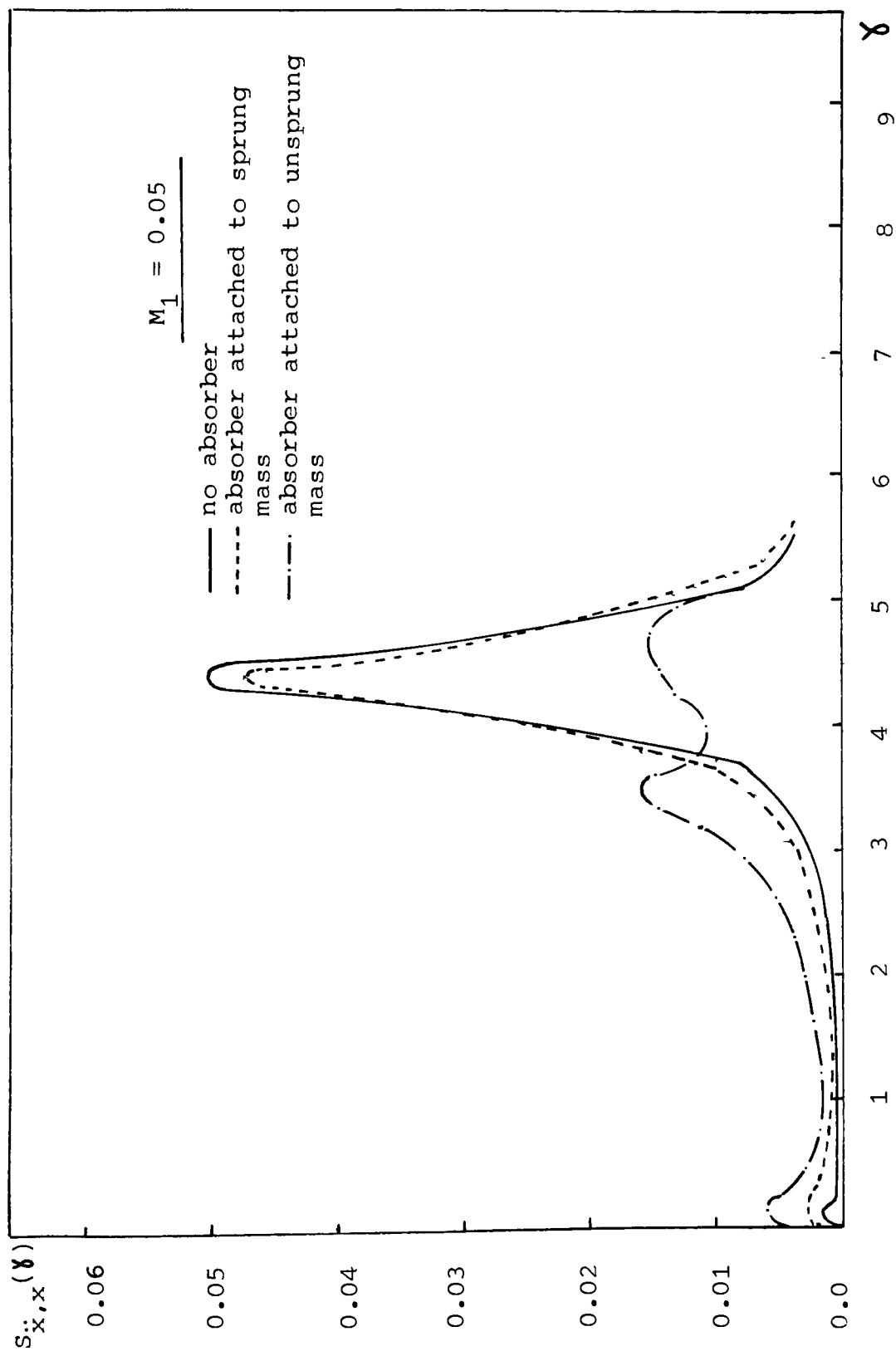


Fig.18. Sprung mass acceleration spectral density when the design parameters are at their constrained optimum values and when, $\ddot{y}_{c,x,rms} = 0.3$

4.2 Optimum Parameters

The values of the constrained optimum f_2^* , f_3^* , K_2^* and K_3^* which optimize the constrained objective function (f_g), for different values of M_1 and M_3 , are plotted against $\ddot{y}_{c'x'rms}$ and shown in Figures 19-21 for the case where the absorber is attached to the unsprung mass, and in Figures 22-24 for the case where the absorber is attached to the sprung mass.

For the case of the optimum damped absorber with the absorber attached to the unsprung mass (Figures 19-21), it is observed that for higher comfort constraints (smaller $\ddot{y}_{c'x'rms}$), higher values of K_3^* and smaller values of K_2^* are required. On the other hand, for the optimum damped absorber with the absorber attached to the sprung mass (Figures 22-24), higher comfort constraints require smaller K_2^* as well as K_3^* . More over, for both cases of optimum damped absorber (Figures 19-24), show that f_2^* and f_3^* play less role than K_2^* and K_3^* . Except for the case where the absorber is attached to the sprung mass and for $M_1 = 0.05$, f_2^* and f_3^* change slightly with $\ddot{y}_{c'x'rms}$.

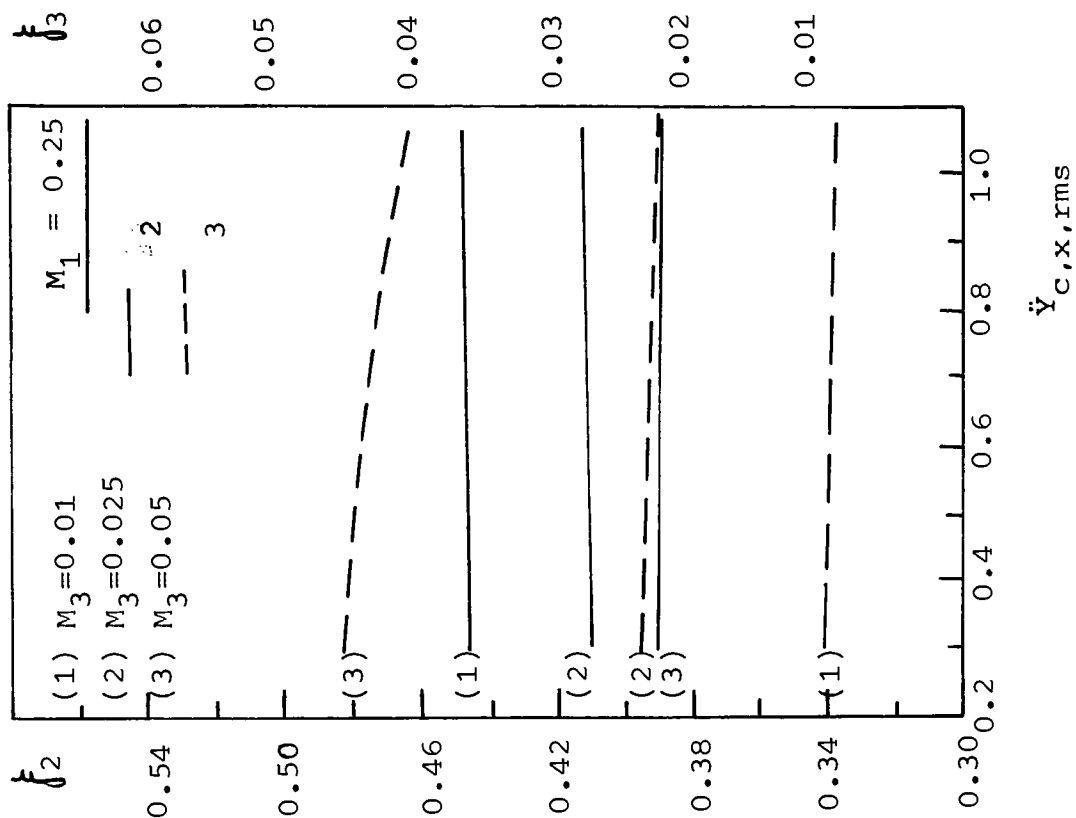
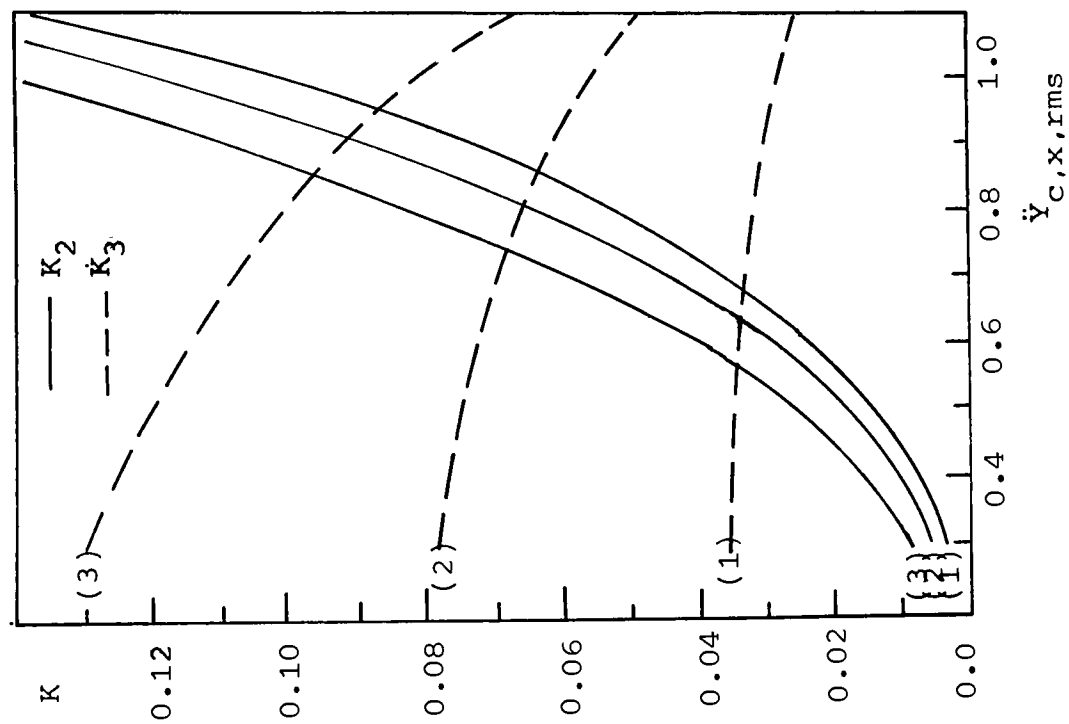


Fig.19 • Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to unsprung mass).

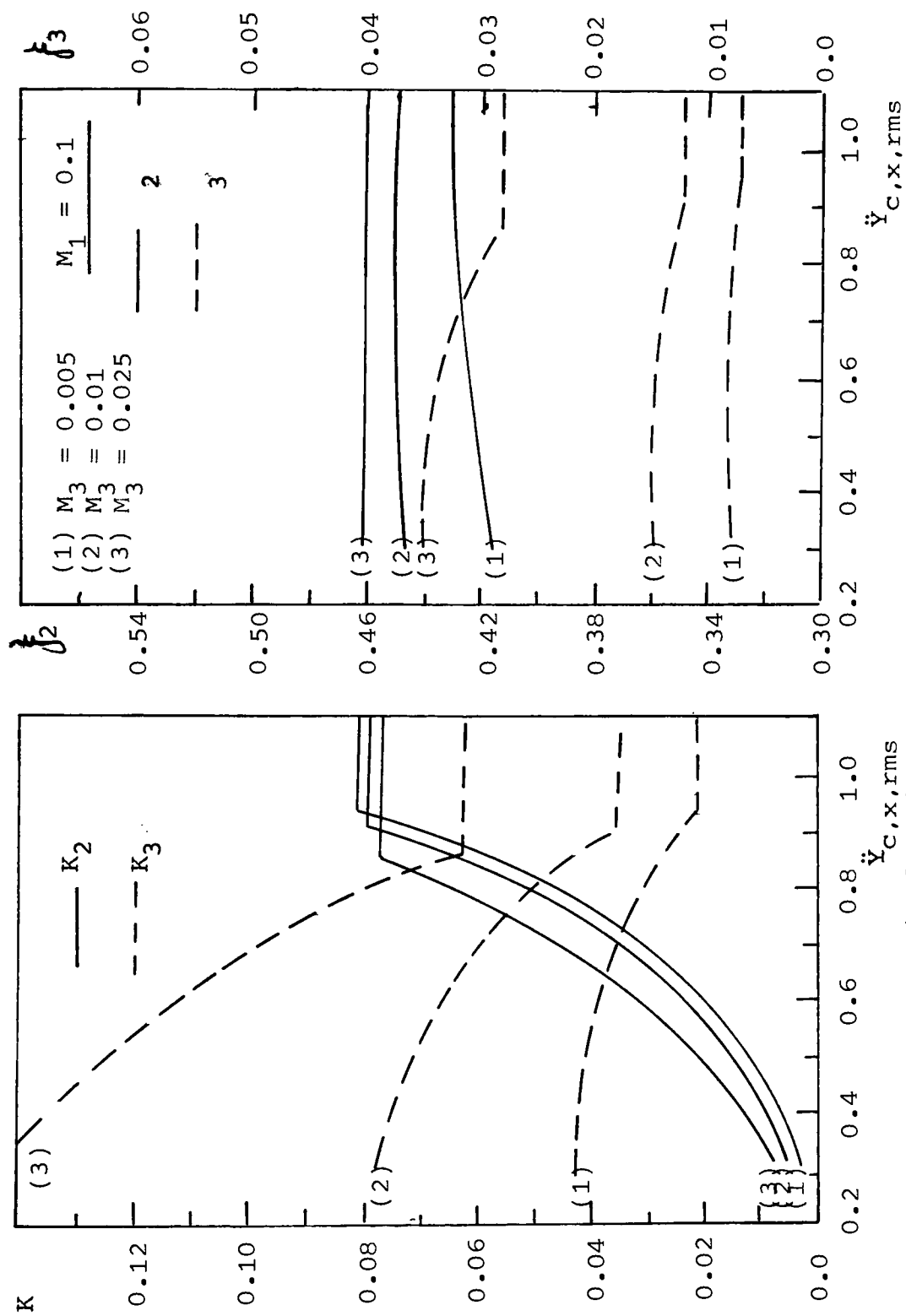


Fig. 20. Constrained optimum parameter values for the constrained optimum damped absorber (damped absorber attached to unsprung mass).

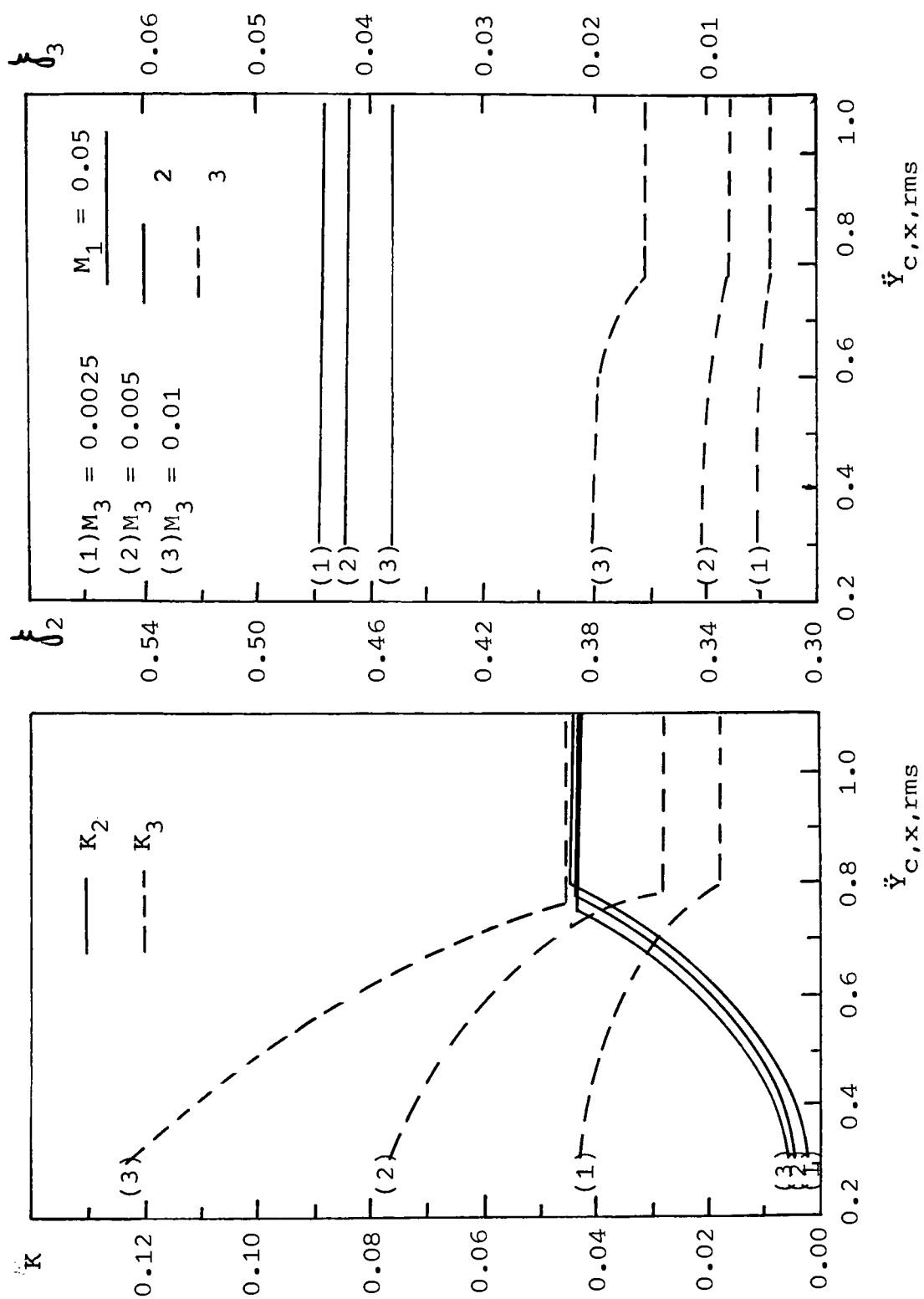


Fig. 21. Constrained optimum parameter values for the constrained damped absorber (absorber attached to unsprung mass).

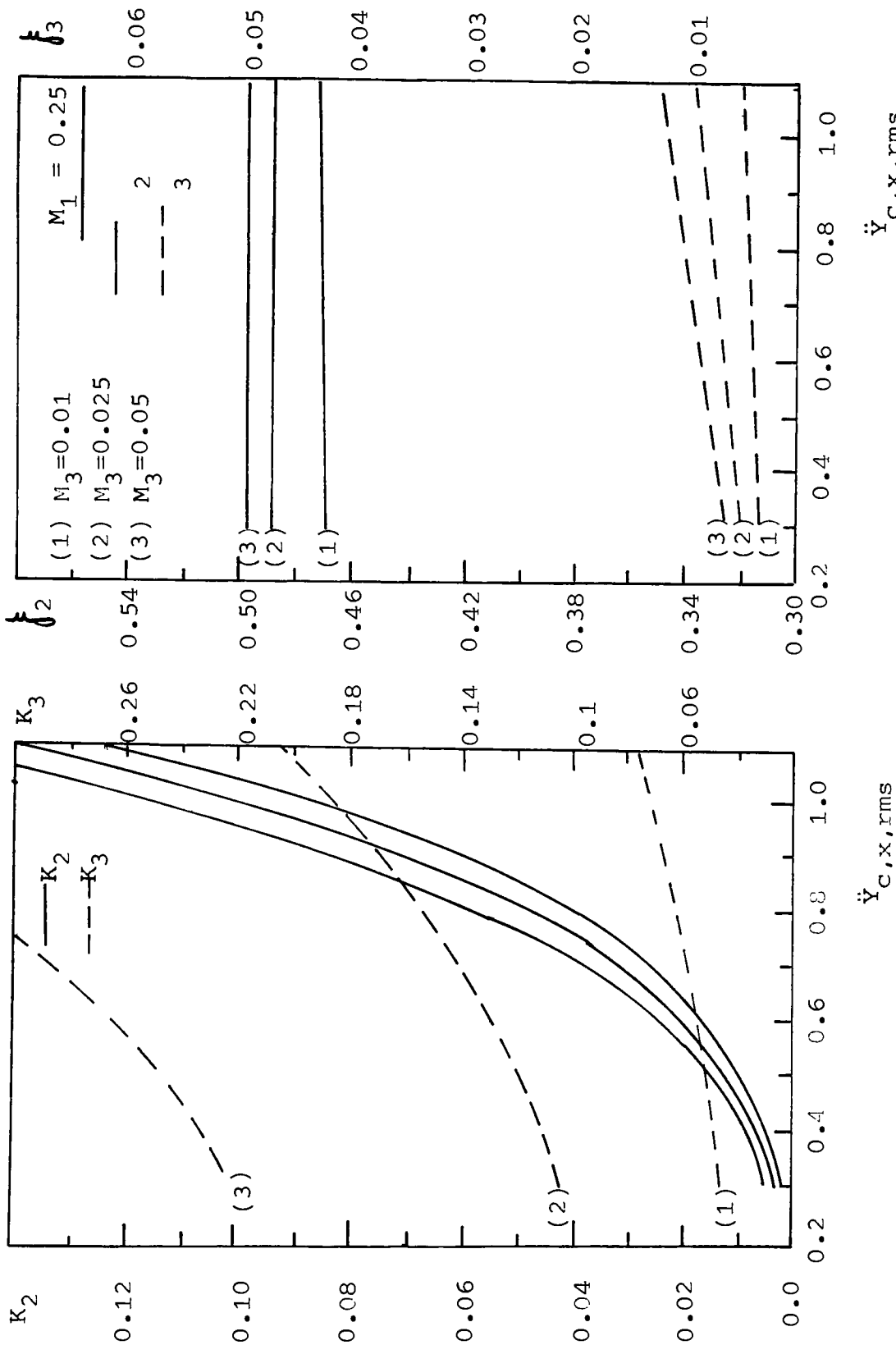


Fig.22. Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass).

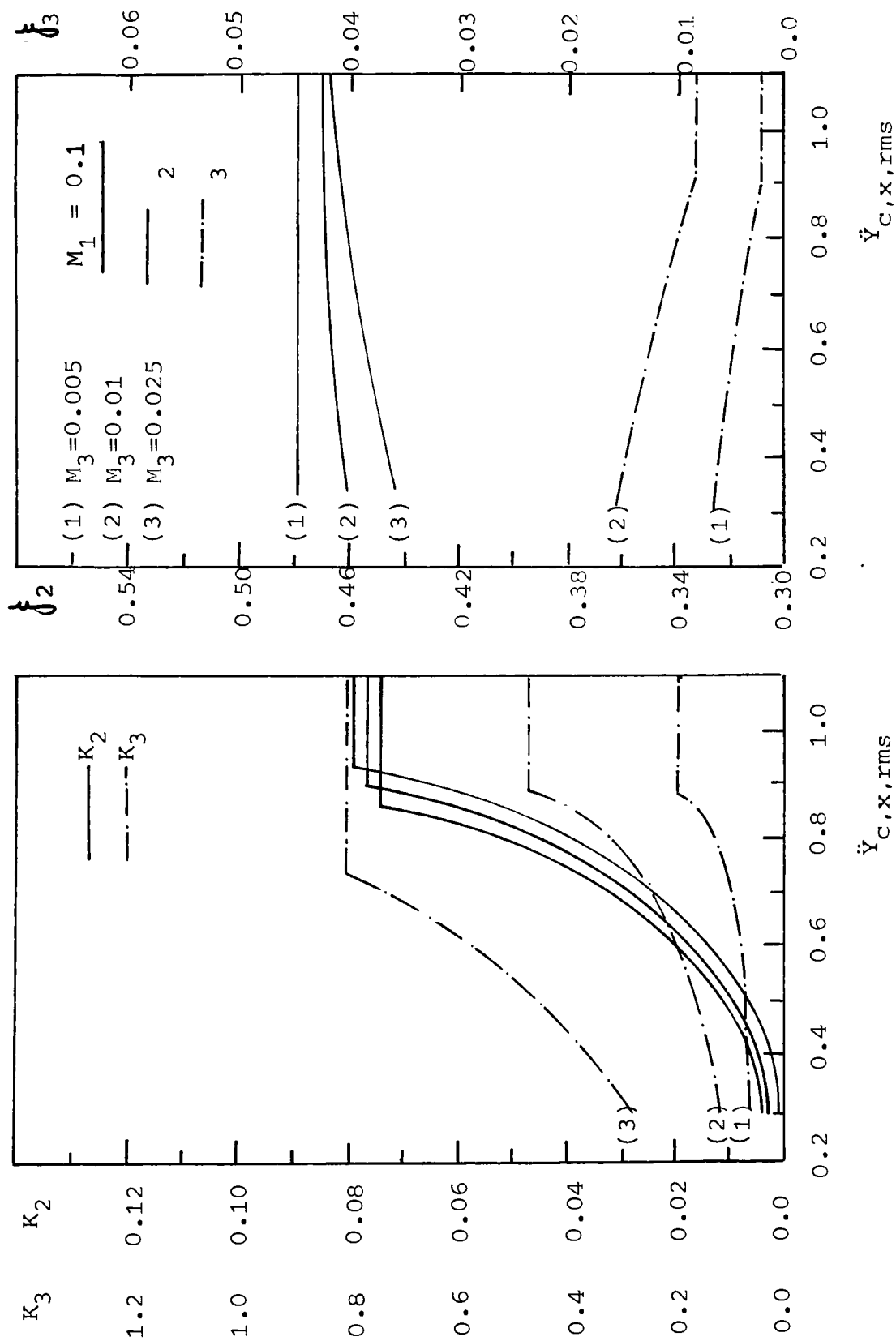


Fig.23. Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass)

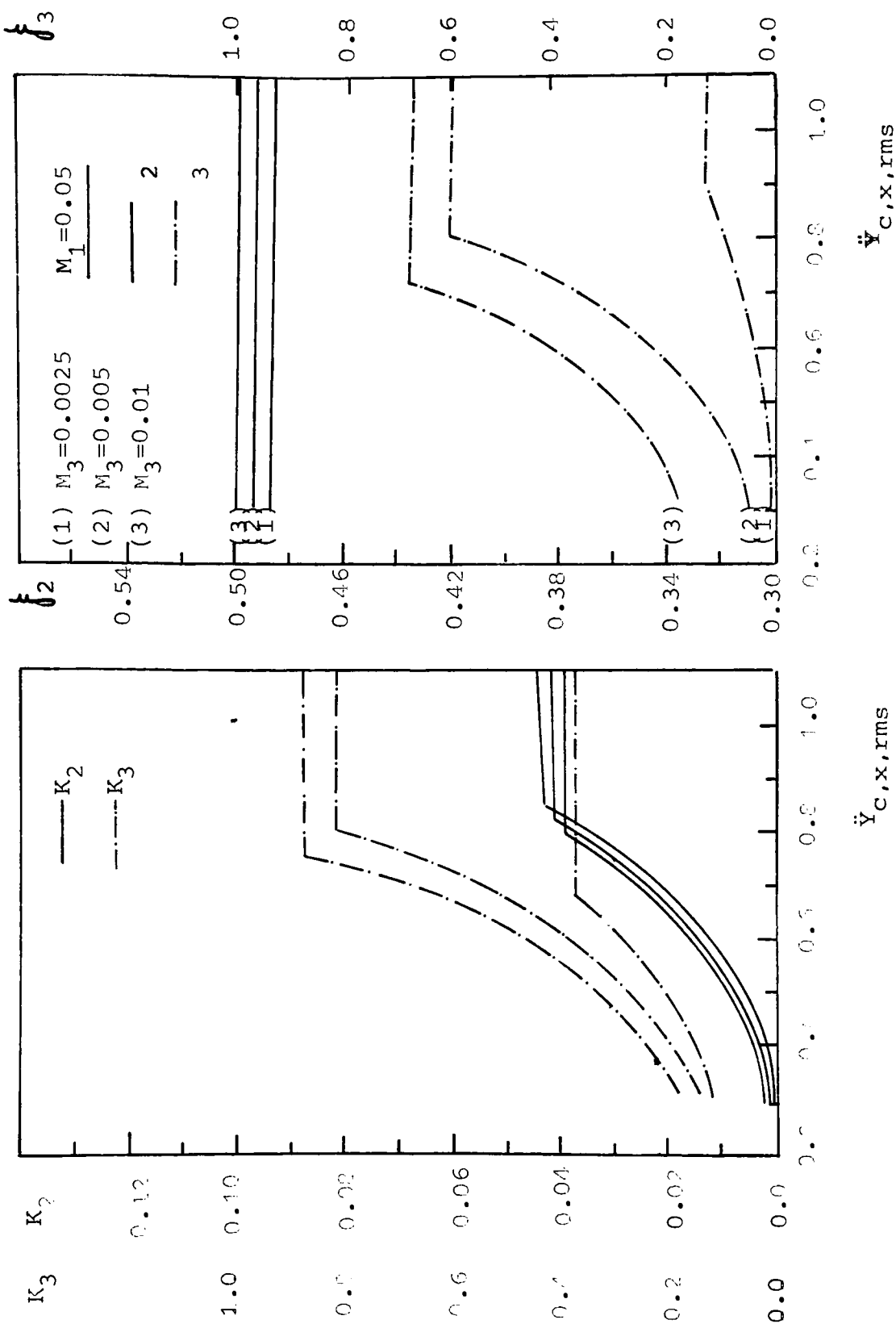


Fig. 24 Constrained optimum parameter values for the constrained optimum damped absorber (absorber attached to sprung mass)

CONCLUSION

This study concludes that, in general, optimum suspension system with damped absorber achieves better performances (ride comfort and controllability) than the optimum conventional suspension, and that the place of attachment of the damped absorber is important.

Mounting the damped absorber to the unsprung mass yields better controllability/ride comfort performances than in the case where the damped absorber is mounted to the sprung mass. Also, attaching the damped absorber to the unsprung mass helps suppress vibration at the second resonant frequency. Damped absorber attached to the sprung mass does not, it slightly reduces the peak of vibration.

The approach adopted in this study can be extended to more sophisticated models and inputs such as higher order systems, non linear systems, and systems subjected to simultaneous guideway and external force disturbances.

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APPENDIX I


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1  REM THIS IS THE MODSER ALGORITHM PROGRAM, P519RE, 9-12-83 VERSION,
   REM MODIFIED FOR USE ON A DIGITAL VAX/VMS COMPUTER
2  REM VARIABLES CHANGED: GT TO GN, AS TO AW, VT( ) TO VU( ), CR TO CF
4  REM THE MODSER ALGORITHM AND PROGRAM P519RE WERE DEVELOPED BY
   REM RAY C. JOHNSON, PHD., P.E.
6  REM SPECIAL PROGRAMMING IS TO BE EMBEDDED IN P519RE AT SUBROUTINES
   REM 18500,19000 AND 20010 IN WHAT FOLLOWS.
8  REM FOR OTHER APPLICATIONS, DELETE THE SPECIAL PROGRAMMING AT
   REM SUBROUTINES 18500, 19000 AND 20010 AND EMBED THE NEW PROBLEM
   REM AT THESE LOCATIONS IN PROPER FORMAT PER MODSER DOCUMENTATION.
10 REM MAIN PROGRAM FOLLOWS
12 DECLARE DOUBLE B,DOUBLE BA,DOUBLE BB,DOUBLE DB,DOUBLE DC,&
   DOUBLE DD,DOUBLE DL
13 DECLARE DOUBLE DS,DOUBLE DT,DOUBLE GM,DOUBLE GN,DOUBLE GU,&
   DOUBLE GV,DOUBLE GW,DOUBLE GX,DOUBLE GY,DOUBLE GZ,&
   DOUBLE H,DOUBLE HA
14 DECLARE DOUBLE HB,DOUBLE OU,DOUBLE OV,DOUBLE OW,DOUBLE P,&
   DOUBLE PA,DOUBLE PG,DOUBLE Q,DOUBLE QA,DOUBLE QB,&
   DOUBLE QD,DOUBLE QE
15 DECLARE DOUBLE QF,DOUBLE QG,DOUBLE RM,DOUBLE TC,DOUBLE TD,&
   DOUBLE TE,DOUBLE TF,DOUBLE TG,DOUBLE TH,DOUBLE TI,&
   DOUBLE TJ,DOUBLE TK,DOUBLE TL
16 DECLARE DOUBLE TM,DOUBLE TN,DOUBLE TP,DOUBLE TQ,DOUBLE W,&
   DOUBLE WA,DOUBLE WB,DOUBLE WD,DOUBLE WE,DOUBLE WF,&
   DOUBLE WK
17 DECLARE DOUBLE X,DOUBLE XA,DOUBLE XB,DOUBLE XD,DOUBLE XE,&
   DOUBLE XF,DOUBLE XG,DOUBLE XI,DOUBLE X3,DOUBLE YJ,&
   DOUBLE YM
19 REM FOR DOUBLE PRECISION, VARIABLES B,D,E,G,H,O-Y SHOULD
   REM BE DBL PREC.
20 Z$ = "MODSER - APPLICATION OF P519RE FOLLOWS (9-12-83 VERSION)"
21 DECLARE DOUBLE DN(25),DOUBLE DP(25),DOUBLE DV(25),DOUBLE E(25),&
   DOUBLE EA(25),DOUBLE EB(25),DOUBLE ES(25),DOUBLE GB(25),&
   DOUBLE GC(25),DOUBLE GS(25),DOUBLE OA(25)
22 DECLARE DOUBLE OD(25),DOUBLE OE(25),DOUBLE OF(25),DOUBLE OG(25),&
   DOUBLE OI(25),DOUBLE OK(25),DOUBLE OS(25)
23 DECLARE DOUBLE R(25),DOUBLE RA(25),DOUBLE RB(25),DOUBLE RS(25),&
   DOUBLE S(25),DOUBLE SA(25),DOUBLE SD(25),DOUBLE SE(25),&
   DOUBLE SF(25),DOUBLE SK(25),DOUBLE SB(25),DOUBLE SC(25),&
   DOUBLE TB(25),DOUBLE U(25),DOUBLE UA(25),DOUBLE UB(25)
24 DECLARE DOUBLE US(25),DOUBLE V(25),DOUBLE VA(25),DOUBLE VB(25),&
   DOUBLE VD(25),DOUBLE VE(25),DOUBLE VF(25),DOUBLE VG(25),&
   DOUBLE VS(25),DOUBLE VU(25),DOUBLE YD(25), DOUBLE YE(25),&
   DOUBLE YF(25),DOUBLE YS(25)
25 PRINT Z$
27 GOSUB 1000
29 GOSUB 8500
35 Z$ = "INITIAL CALCULATIONS:"
37 PRINT Z$
39 PRINT
50 FOR I = 1 TO NV
51 CD(I)=(CX(I)-CN(I))/AL
53 NEXT I
65 A = 0.
67 FOR I = 1 TO NV
69 A = A + (CX(I) - CN(I)) ^ 2
71 NEXT I
73 AD = SQR (A) / AB
75 FD = AB * AD / AW
77 FS = FD / 5
78 FX = FS / AT
79 FH = AD / 2

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80 IF NE = 0 AND NR = 0 THEN 81 ELSE 95
81 FM = 0
82 GOTO 110
95 FM = AW / AE / AB
110 Z$ = "AD,FD,FS,FX,FH,FM="
112 PRINT Z$,AD,FD,FS,FX,FH,FM
114 GOSUB 1015
115 GOSUB 1015
116 FS = 100 * FS
117 FX = 100 * FX
118 ME = 0
125 IF MX > 0 THEN GOSUB 2000
140 K = 1
141 KT = 1
142 KE = 0
143 MF = 1
144 MG = 0
145 MR = 0
146 NN = 0
147 KP = 0
148 FP = 1.
149 DC = 0.0E0
150 KA = 0
151 NX = 0
152 DL = AD
153 IT = 0
155 FOR I = 1 TO NV
156 VB(I) = CS(I)
157 V(I) = VB(I)
158 NEXT I
159 IF MS = 0 THEN 160 ELSE 170
160 GOSUB 9000
161 GOTO 200
170 REM FOR DBL PREC, IN LINE 80, VB(I) SHOULD = CDBL(CS(I))
185 FOR IS = 1 TO MS
186 OK(IS) = OS(IS)
187 OI(IS) = OK(IS)
188 NEXT IS
189 GOSUB 9000
200 IF NU > 0 THEN 201 ELSE 215
201 FOR IU = 1 TO NU
202 UB(IU) = U(IU)
203 NEXT IU
215 IF MS = 0 THEN 245
230 FOR IS = 1 TO MS
231 SK(IS) = S(IS)
232 NEXT IS
245 IF KG > 0 THEN 246 ELSE 260
246 FOR JG = 1 TO KG
247 NB(JG) = N(JG)
248 NEXT JG
260 IF NE > 0 THEN 261 ELSE 275
261 FOR IE = 1 TO NE
262 EB(IE) = E(IE)
263 NEXT IE
275 IF NR > 0 THEN 276 ELSE 290
276 FOR IR = 1 TO NR
277 RB(IR) = R(IR)
278 NEXT IR
290 QB = Q
291 BB = B
292 HB = H
293 WB = W

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294 PB = P
295 XB = X
296 GOSUB 1000
297 Z$ = "START OF MODSER SEARCH PROCESS:"
298 PRINT Z$
299 GOSUB 1015
300 Z$ = "BASE POINT PRINTOUTS FOLLOW:"
301 PRINT Z$
320 GOSUB 1030
321 GOSUB 1015
322 IF KT = 1 THEN 323 ELSE 335
323 GOSUB 8240
324 GOTO 350
335 ON NP GOSUB 8195,8240
350 GOSUB 3000
365 GOSUB 5000
366 FE = FR
380 Z$ = "JT,FB,FE="
381 PRINT Z$;JT,FB,FE
395 XI = 0.0E0
396 FOR I = 1 TO NV
397 GX = VA(I) - VB(I)
398 XI = XI + GX * GX
399 NEXT I
400 X3=SQR(XI)
401 DB = X3
402 PRINT "DB=";DB
403 REM DB SHOULD BE SINGLE PREC.
410 IF KA = 1 THEN 411 ELSE 425
411 DL = DB
412 KA = 0
425 IF DB > FS THEN 500
440 IF ME = 3 THEN 530
455 IF ME = 2 THEN 456 ELSE 470
456 ME = 3
457 GOTO 530
470 IF ME = 0 THEN 471 ELSE 485
471 ME = 1
472 FX = FX / 10
473 FS = FS / 10
474 GOTO 530
485 IF ME = 1 THEN 486 ELSE 500
486 ME = 2
487 FX = FX / 10
488 FS = FS / 10
489 GOTO 530
500 IF KT >= KX THEN 1075
515 GOSUB 1225
516 K = K + 1
517 KT = KT + 1
518 GOTO 320
530 PRINT "ME=";ME
531 IF CP = 0. THEN 560
545 IF FP < FM THEN 590
560 IF K = 1 AND ME = 3 THEN 680
575 K = 0
576 IF CP = 0. THEN 500
590 FP = AM * FP
591 Z$ = "PENALTY INCREASED AT 590,WITH FP="
592 PRINT Z$;FP
593 GOSUB 1030
605 FOR I = 1 TO NV
606 V(I) = VA(I)

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607 NEXT I
608 IF MS > 0 THEN 609 ELSE 620
609 FOR IS = 1 TO MS
610 OI(IS) = OA(IS)
611 NEXT IS
620 GOSUB 9000
621 BA = B
622 HA = H
623 WA = W
624 PA = P
625 XA = X
635 IF NE > 0 THEN 636 ELSE 650
636 FOR IE = 1 TO NE
637 EA(IE) = E(IE)
638 NEXT IE
650 IF NR > 0 THEN 651 ELSE 665
651 FOR IR = 1 TO NR
652 RA(IR) = R(IR)
653 NEXT IR
665 KA = 1
666 GOTO 500
680 KE = KE + 1
681 Z$ = "TERMINATION TEST COUNT KE="
682 PRINT Z$;KE
683 GOSUB 1030
695 GX = DB + DC
696 IF GX > 0.0E0 THEN 697 ELSE 710
697 GY = DC + DD
698 AC = GY / GX
699 GOTO 725
710 AC = 1.E20
725 IF AC <= 1 THEN 726 ELSE 740
726 Z$ = "DIVERGENCE ANTICIPATED,WITH AC="
727 PRINT Z$;AC
728 Z$ = "WENT TO 575 FOR NEXT LOOP"
729 PRINT Z$
730 GOSUB 1030
731 GOTO 575
740 Z$ = "CONVERGENCE ANTICIPATED,WITH AC="
741 PRINT Z$;AC
742 FT = (AC - 1) * 6 * FS
743 Z$ = "READY FOR TERMINATION TEST AT 755, AND FT="
744 PRINT Z$;FT
755 IF DB > FT THEN 756 ELSE 770
756 Z$ = "TERMINATION TEST FAILED AT 755. WENT TO 575 &
      FOR NEXT LOOP."
757 PRINT Z$
758 GOSUB 1030
759 GOTO 575
770 Z$ = "TERMINATION TEST PASSED AT 755"
771 PRINT Z$
785 GOSUB 1000
786 GOSUB 1000
787 Z$ = "SOLUTION FOUND IS AT (K+1), NOW TRANSFERRED TO &
      (K) STORAGE FOR PRINTOUT"
788 PRINT Z$
789 PRINT
800 KK = 0
801 GOSUB 1225
802 GOSUB 4000
803 XI = 0.0E0
804 FOR I = 1 TO NV
805 XI = XI + CB(I) + CB(I)

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806 NEXT I
807 X3=SQR(XI)
808 GM = X3
809 GOSUB 8240
815 PRINT "GM=";GM
816 GOSUB 1015
830 Z$ = "DOUBLE PRECISION VB(I) FOR I=1,...,NV ARE="
831 PRINT Z$
832 FOR I = 1 TO NV
833 PRINT USING "#####.#####", VB(I),
834 NEXT I
835 PRINT
836 IF NU = 0 THEN 860
845 Z$ = "DOUBLE PRECISION UB(IU) FOR IU=1,...,NU ARE ="
846 PRINT Z$
847 FOR IU = 1 TO NU
848 PRINT USING "#####.#####", UB(IU),
849 NEXT IU
850 PRINT
860 GOSUB 1015
861 FOR I = 1 TO NV
862 V(I) = VB(I)
863 NEXT I
864 IF MS > 0 THEN 865 ELSE 875
865 FOR IS = 1 TO MS
866 OI(IS) = OK(IS)
867 NEXT IS
875 GOSUB 9000
876 GOSUB 10000
877 PRINT
878 GOSUB 1000
890 Z$ = "MF,MG,MR,NX="
891 PRINT Z$;MF,MG,MR,NX
892 CG(11)=CG(11)+.1
893 IF CG(11) < 1.2 GO TO 35
905 GOTO 32000
995 REM
998 REM ***MINOR SUBROUTINES***
999 REM
1000 FOR I=1 TO 72
1001 PRINT "*";
1002 NEXT I
1003 PRINT
1004 RETURN
1015 FOR I = 1 TO 70
1016 PRINT "-";
1017 NEXT I
1018 PRINT
1019 RETURN
1030 FOR I = 1 TO 25
1031 PRINT "-";
1032 NEXT I
1033 PRINT
1034 RETURN
1035 REM
1036 REM
1045 DIM C(KC),CD(NV),CG(NG),CN(NV),CS(NV),CX(NV),F(NF)
1047 DIM N(KG),NA(KG),NB(KG),NC(KG),NS(KG)
1062 RETURN
1063 REM
1064 REM
1075 Z$ = "KT=KX STOP REVIEW;BASE POINT PRINTOUT:"
1076 GOSUB 1015

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1077 GOSUB 1015
1078 PRINT Z$
1079 GOSUB 8240
1080 FOR M = 1 TO 3
1081 PRINT
1082 NEXT M
1083 PRINT "ENTER '1' FOR NEXT LOOP, '2' TO END"
1084 INPUT ZZ%
1085 ON ZZ% GOTO 515,32000
1086 REM
1087 REM
1090 REM    **NEW BASE POINT ANALYSIS SUBROUTINE 1105:**
1092 REM
1105 FOR I = 1 TO NV
1106 V(I) = VB(I)
1107 NEXT I
1108 IF MS > 0 THEN GOTO 1109 ELSE 1120
1109 FOR IS = 1 TO MS
1110 OI(IS) = OK(IS)
1111 NEXT IS
1120 GOSUB 9000
1121 QB = Q
1122 BB = B
1123 HB = H
1124 WB = W
1125 PB = P
1126 XB = X
1127 IF NU > 0 THEN GOTO 1128 ELSE 1135
1128 FOR IU = 1 TO NU
1129 UB(IU) = U(IU)
1130 NEXT IU
1135 IF MS > 0 THEN GOTO 1136 ELSE 1150
1136 FOR IS = 1 TO MS
1137 SK(IS) = S(IS)
1138 NEXT IS
1150 IF NE > 0 THEN GOTO 1151 ELSE 1165
1151 FOR IE = 1 TO NE
1152 EB(IE) = E(IE)
1153 NEXT IE
1165 IF NR > 0 THEN GOTO 1166 ELSE 1180
1166 FOR IR = 1 TO NR
1167 RB(IR) = R(IR)
1168 NEXT IR
1180 IF KG > 0 THEN GOTO 1181 ELSE 1195
1181 FOR JG = 1 TO KG
1182 NB(JG) = N(JG)
1183 NEXT JG
1195 RETURN
1200 REM
1202 REM
1210 REM    **MAIN PROGRAM (K) STORAGE INDEX SUBROUTINE 1225**
1212 REM
1225 DD = DC
1226 DC = DB
1227 FOR I = 1 TO NV
1228 GC(I) = GB(I)
1229 SC(I) = SB(I)
1230 NEXT I
1240 QB = QA
1241 BB = BA
1242 HB = HA
1243 WB = WA
1244 PB = PA

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```

1245 XB = XA
1246 FOR I = 1 TO NV
1247 VB(I) = VA(I)
1248 NEXT I
1250 IF MS > 0 THEN GOTO 1251 ELSE 1255
1251 FOR IS = 1 TO MS
1252 OK(IS) = OA(IS)
1253 SK(IS) = SA(IS)
1254 NEXT IS
1255 IF NU > 0 THEN 1256 ELSE 1270
1256 FOR IU = 1 TO NU
1257 UB(IU) = UA(IU)
1258 NEXT IU
1270 IF NE > 0 THEN 1271 ELSE 1285
1271 FOR IE = 1 TO NE
1272 EB(IE) = EA(IE)
1273 NEXT IE
1285 IF NR > 0 THEN GOTO 1286 ELSE 1300
1286 FOR IR = 1 TO NR
1287 RB(IR) = RA(IR)
1288 NEXT IR
1300 IF KG > 0 THEN GOTO 1301 ELSE 1315
1301 FOR JG = 1 TO KG
1302 NC(JG) = NB(JG)
1303 NB(JG) = NA(JG)
1304 NEXT JG
1315 RETURN
1320 REM
1330 REM
1998 REM ***SUBROUTINE 2000:SG SEARCH***
2000 KS = 1
2001 XG = 1.E19
2002 CP = AG
2003 FP = 1.
2005 PRINT "SG SEARCH FOLLOWS:"
2014 FOR I = 1 TO NV
2015 RM=RND
2016 V(I) = CN(I) + RM * (CX(I) - CN(I))
2017 NEXT I
2018 GOSUB 9000
2019 PRINT "KS,V(I),P,Q="
2020 PRINT KS,V(1),P,Q
2021 PRINT
2030 IF X < XG THEN 2031 ELSE 2045
2031 XG = X
2032 QG = Q
2033 PG = P
2034 FOR I = 1 TO NV
2035 VG(I) = V(I)
2036 NEXT I
2045 IF KS < MN THEN 2075
2060 IF PG = 0 THEN 2090
2075 IF KS < MX THEN 2076 ELSE 2090
2076 KS = KS + 1
2077 GOTO 2014
2090 Z$ = "SG SEARCH GAVE FOLLOWING POINT:"
2091 PRINT
2092 PRINT
2093 PRINT Z$
2094 PRINT
2095 Z$ = "QG,PG,XG="
2096 PRINT Z$,QG,PG,XG
2097 PRINT

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```

2105 Z$ = "VG(I) FOR I=1,...,NV ARE="
2106 PRINT Z$
2107 FOR I = 1 TO NV
2108 PRINT VG(I),
2109 NEXT I
2110 PRINT
2135 FOR I = 1 TO NV
2136 CS(I) = VG(I)
2137 NEXT I
2138 RETURN
2998 REM ***SUBROUTINE 3000:SEARCH DIRECTION SB(I),TB(I),
    REM AT VB(I);FOR I=1 TO NV***
3000 Z$ = "SUBROUTINE 3000 STARTED - FOR SEARCH DIRECTION &
    AT BASE POINT (K)"
3001 REM PRINT Z$
3002 GOSUB 4000
3015 IF ND = 1 OR DC = 0. THEN 3135
3030 IF KA > 0 THEN 3135
3045 IF ND = 2 OR KG = 0 THEN 3150
3060 JG = 1
3075 IF NB(JG) <> NC(JG) THEN 3120
3090 IF JG = KG THEN 3150
3105 JG = JG + 1
3106 GOTO 3075
3120 NX = NX + 1
3121 Z$ = "NX,JG="
3122 PRINT Z$,NX,JG
3135 FOR I = 1 TO NV
3136 SB(I) = - GB(I)
3137 NEXT I
3138 ID = 1
3139 GOTO 3195
3150 GX = 0.0E0
3151 GY = GX
3165 FOR I = 1 TO NV
3166 GX = GX + (GB(I) - GC(I)) * GB(I)
3167 GY = GY + (GB(I) - GC(I)) * SC(I)
3168 NEXT I
3169 IF GY = 0. THEN 3170 ELSE 3180
3170 Z$ = "GY=0. AT 3165; GOTO 3135"
3171 PRINT Z$
3172 GOTO 3135
3180 WK = GX / GY
3181 FOR I = 1 TO NV
3182 SB(I) = - GB(I) + WK * SC(I)
3183 NEXT I
3184 ID = 2
3195 PRINT "ID=";ID
3210 DT = 0.0E0
3211 FOR I = 1 TO NV
3212 DT = DT + GB(I) * SB(I)
3213 NEXT I
3214 IF DT < 0. THEN 3270
3225 IF DT = 0. AND ID = 1 THEN 3226 ELSE 3240
3226 Z$ = "STATIONARY POINT FOUND AT 3225,WITH ID,DT="
3227 PRINT Z$;ID,DT
3228 RETURN
3240 IF ID = 2 THEN 3241 ELSE 3255
3241 MR = MR + 1
3242 GOTO 3135
3255 Z$ = "FAILED DESCENT DIRECTION TEST AT 3240,WITH GX="
3256 PRINT Z$;GX
3257 GOTO 32000

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3270 XI = 0.0E0
3271 FOR I = 1 TO NV
3272 XI = XI + SB(I) * SB(I)
3273 NEXT I
3274 X3=SQR(XI)
3285 FOR I = 1 TO NV
3286 IF X3 > 0.0E0 THEN 3287 ELSE 3300
3287 TB(I) = SB(I) / X3
3288 GOTO 3315
3300 TB(I) = 0.0E0
3315 NEXT I
3316 RETURN
3998 REM ***SUBROUTINE 4000: GRADIENT GB(I) AT BASE POINT
      REM      VB(I);FOR I=1 TO NV***
4000 MG = MG + 1
4001 FOR I = 1 TO NV
4002 V(I) = VB(I)
4003 NEXT I
4004 IF MS = 0 THEN 4030
4015 FOR IS = 1 TO MS
4016 OI(IS) = SK(IS)
4017 NEXT IS
4030 FOR I = 1 TO NV
4045 VU(I) = V(I)
4060 V(I) = VB(I) + CD(I)
4061 GOSUB 9000
4062 DP(I) = X
4075 V(I) = VB(I) - CD(I)
4076 GOSUB 9000
4077 DN(I) = X
4090 GB(I) = (DP(I) - DN(I)) / 2 / CD(I)
4105 V(I) = VU(I)
4120 NEXT I
4135 RETURN
4998 REM ***SUBROUTINE 5000:LINE SEARCH FOR (K+1)
      REM      BASE POINT - VA(I),I=1 TO NV;AND CP TUNING
      REM      CALCULATIONS***
5000 Z$ = "SUBROUTINE 5000 STARTED - LINE SEARCH FOR V AT&
      (K+1);AND CP TUNING"
5001 REM PRINT Z$
5015 IF DT = 0. AND ID = 1 AND KT > 1. AND CP <> 0. THEN&
      5016 ELSE 5030
5016 FR = 0.
5017 FB = 0.
5018 J = 0
5019 GOTO 5375
5030 IF KT = 1 THEN 5031 ELSE 5045
5031 FR = AD
5032 GOTO 5105
5045 IF KA > 0 THEN 5046 ELSE 5060
5046 FR = CL * DL
5047 GOTO 5105
5060 IF DC = 0. THEN 5061 ELSE 5075
5061 FR = FS / 2
5062 GOTO 5105
5075 IF DC < AD THEN 5076 ELSE 5090
5076 FR = CA * DC
5077 GOTO 5105
5090 FR = AD
5105 FB = FR
5120 FOR I = 1 TO NV
5121 DV(I) = FR * TB(I)
5122 NEXT I

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```

5135 JT = 0
5136 J = 0
5137 QD = QB
5138 WD = WB
5139 XD = XB
5140 FOR I = 1 TO NV
5141 YD(I) = 0.0E0
5142 VD(I) = VB(I)
5143 NEXT I
5144 IF MS = 0 THEN 5165
5150 FOR IS = 1 TO MS
5151 OG(IS) = SK(IS)
5152 OD(IS) = OK(IS)
5153 SD(IS) = SK(IS)
5154 NEXT IS
5165 QF = QE
5166 WF = WE
5167 XF = XE
5168 FOR I = 1 TO NV
5169 YF(I) = YE(I)
5170 VF(I) = VE(I)
5171 NEXT I
5172 IF MS = 0 THEN 5195
5180 FOR IS = 1 TO MS
5181 OF(IS) = OE(IS)
5182 SF(IS) = SE(IS)
5183 NEXT IS
5195 QE = QD
5196 WE = WD
5197 XE = XD
5198 FOR I = 1 TO NV
5199 YE(I) = YD(I)
5200 VE(I) = VD(I)
5201 NEXT I
5202 IF MS = 0 THEN 5225
5210 FOR IS = 1 TO MS
5211 OE(IS) = OD(IS)
5212 SE(IS) = SD(IS)
5213 NEXT IS
5225 JT = JT + 1
5226 J = J + 1
5227 REM PRINT "JT,J=";JT,J
5240 FOR I = 1 TO NV
5241 YD(I) = YE(I) + DV(I)
5242 VD(I) = VB(I) + YD(I)
5243 V(I) = VD(I)
5244 NEXT I
5245 IF MS = 0 THEN 5270
5255 FOR IS = 1 TO MS
5256 OD(IS) = OG(IS)
5257 OI(IS) = OD(IS)
5258 NEXT IS
5270 GOSUB 9000
5271 IF MS = 0 THEN 5300
5285 FOR IS = 1 TO MS
5286 SD(IS) = S(IS)
5287 OG(IS) = SD(IS)
5288 NEXT IS
5300 MF = MF + 1
5301 QD = Q
5302 WD = W
5303 XD = X
5315 IF CP = 0. AND WD >= WE AND WD > 0.THEN 5945

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```

5330 NN = 0
5345 IF J >= 2 THEN 5420
5360 IF XD <= XE THEN 5361 ELSE 5375
5361 MC = 1
5362 GOTO 5165
5375 GZ = FX * CF
5376 IF FR > GZ THEN 5405
5377 FOR I = 1 TO NV
5378 VA(I) = VD(I)
5379 V(I) = VA(I)
5380 NEXT I
5381 IF MS = 0 THEN 5555
5390 FOR IS = 1 TO MS
5391 OA(IS) = OD(IS)
5392 OI(IS) = OA(IS)
5393 NEXT IS
5394 GOTO 5555
5405 J = 0
5406 FR = CF * FR
5407 FOR I = 1 TO NV
5408 DV(I) = CF * DV(I)
5409 NEXT I
5410 GOTO 5225
5420 IF XD < XE THEN 5855
5435 GX = XD + XF - 2 * XE
5436 IF GX = 0 THEN 5437 ELSE 5450
5437 DS = FR
5438 GOTO 5465
5450 DS = FR * (XD - XE) / GX
5465 XI = 0.0E0
5466 FOR I = 1 TO NV
5467 XI = XI + YD(I) * YD(I)
5468 NEXT I
5469 X3=SQR(XI)
5470 YJ = X3
5471 YM = YJ - FR / 2 - DS
5480 IF YM < 0.0E0 THEN 5481 ELSE 5495
5481 Z$ = "FAILED TEST AT 5480"
5482 PRINT Z$
5483 GOTO 5795
5495 FOR I = 1 TO NV
5496 YS(I) = YM * TB(I)
5497 VA(I) = VB(I) + YS(I)
5498 V(I) = VA(I)
5499 NEXT I
5500 GN = DS + FR / 2
5501 GU = FR * FR
5502 GV = 2 * FR
5510 IF MS = 0 THEN 5555
5525 FOR IS = 1 TO MS
5540 OU = OD(IS)
5541 OV = OE(IS)
5542 OW = OF(IS)
5543 GOSUB 5570
5544 OI(IS) = GZ
5545 NEXT IS
5555 GOSUB 9000
5556 GOTO 5585
5570 GW = OV - OU
5571 GX = OW - OU
5572 GY = (4 * GW - GX) / GV
5573 GX = (GW - GY * FR) / GU
5574 GW = GN * (GX * GN + GY)

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```

5575 GZ = GW + OU
5576 RETURN
5585 MF = MF + 1
5586 QA = Q
5587 BA = B
5588 HA = H
5589 WA = W
5590 PA = P
5591 XA = X
5600 IF NU > 0 THEN 5601 ELSE 5615
5601 FOR IU = 1 TO NU
5602 UA(IU) = U(IU)
5603 NEXT IU
5615 IF MS = 0 THEN 5645
5630 FOR IS = 1 TO MS
5631 OA(IS) = OI(IS)
5634 SA(IS) = S(IS)
5637 NEXT IS
5645 IF NE > 0 THEN 5647 ELSE 5660
5647 FOR IE = 1 TO NE
5648 EA(IE) = E(IE)
5651 NEXT IE
5660 IF NR>0 THEN 5661 ELSE 5675
5661 FOR IR = 1 TO NR
5663 RA(IR) = R(IR)
5666 NEXT IR
5675 IF KG > 0 THEN 5676 ELSE 5690
5676 FOR JG = 1 TO KG
5678 NA(JG) = N(JG)
5681 NEXT JG
5690 IF XA > XB THEN 5720
5705 RETURN
5720 Z$ = "FAILED FUNCTION DECREASE TEST AT 5690"
5723 PRINT Z$
5735 GZ = FX * CF
5738 IF FR > GZ THEN 5739 ELSE 5750
5739 PRINT "WENT TO 5825 AND 5405"
5741 GOTO 5825
5750 RETURN
5765 GZ = FX * CF
5768 IF FR > GZ THEN 5769 ELSE 5780
5769 PRINT "WENT TO 5823 FROM 5765"
5771 GOTO 5825
5780 IF ABS (YM) > GZ THEN 5781 ELSE 5795
5781 PRINT "WENT TO 5825 FROM 5780"
5783 GOTO 5825
5795 FOR I = 1 TO NV
5799 VA(I) = VB(I)
5802 V(I) = VA(I)
5805 NEXT I
5807 IF MS = 0 THEN 5555
5810 FOR IS = 1 TO MS
5813 OA(IS) = OK(IS)
5816 OI(IS) = OA(IS)
5819 NEXT IS
5820 GOTO 5555
5825 QE = QF
5826 WE = WF
5827 XE = XF
5828 FOR I = 1 TO NV
5829 YE(I) = YF(I)
5830 VE(I) = VF(I)
5831 NEXT I

```

```

5832 IF MS > 0 THEN 5833 ELSE 5840
5833 FOR IS = 1 TO MS
5834 OE(IS) = OF(IS)
5835 SE(IS) = SF(IS)
5836 NEXT IS
5840 GOTO 5405
5855 IF J > JX THEN 5856 ELSE 5870
5856 Z$ = "J>JX AT 5855;STOP REVIEW"
5857 GOSUB 1000
5858 PRINT Z$
5859 GOSUB 8240
5860 GOTO 32000
5870 IF FR > FH THEN 5165
5885 IF MC < 1 THEN 5886 ELSE 5900
5886 MC = 1
5887 GOTO 5165
5900 FR = 2 * FR
5901 QE = QF
5902 WE = WF
5903 XE = XF
5904 FOR I = 1 TO NV
5905 DV(I) = 2 * DV(I)
5906 YE(I) = YF(I)
5907 VE(I) = VF(I)
5908 NEXT I
5909 IF MS = 0 THEN 5910 ELSE 5915
5910 MC = 0
5911 GOTO 5930
5915 FOR IS = 1 TO MS
5916 OE(IS) = OF(IS)
5917 SE(IS) = SF(IS)
5918 NEXT IS
5919 MC = 0
5930 GOTO 5165
5945 IF KP > 0 THEN 5975
5960 IF QD > QE THEN 5961 ELSE 5975
5961 NN = 0
5962 Z$ = "IN PENALTY ZONE AT 5945 WITH WD>WE,BUT NO &
      TUNE SINCE QD>QE"
5963 PRINT Z$
5964 GOTO 5345
5975 IF NN = 0 THEN 6035 ELSE 5990
5976 REM ***AT FIRST PROBE EDGE OF PENALTY ZONE***
5990 IF KP = 0 THEN 5991 ELSE 6005
5991 KP = 1
5992 GOTO 5165
6005 KP = 2
6006 GOTO 6095
6007 REM ***CP CALCULATION***
6020 REM **SEGMENT 6035: FOR CP SELF-TUNING PROCESS**
6035 Z$ = "START SEGMENT 6035 - AT EDGE OF PENALTY &
      ZONE WITH WD>WE AND QD<QE"
6036 PRINT Z$
6050 FOR I = 1 TO NV
6051 VB(I) = VE(I)
6052 NEXT I
6053 IF MS = 0 THEN 6080
6065 FOR IS = 1 TO MS
6066 OK(IS) = OE(IS)
6067 NEXT IS
6080 GOSUB 1105
6081 NT = ND
6082 ND = 1

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```

6083 GOSUB 3000
6084 FB = FR
6085 NN = 1
6086 KP = 0
6087 ND = NT
6088 GOTO 5120
6095 Z$ = "READY FOR CP CALCULATION AT 6095"
6096 PRINT Z$
6110 IF WD <> WF THEN CP = .75 * (QF - QD) / (WD - WF)
6125 Z$ = "AT (J-2), AND KP=0: QF,WF="
6126 PRINT Z$;QF,WF
6140 Z$ = "AT (J-1), AND KP=1:QE,WE="
6141 PRINT Z$;QE,WE
6155 Z$ = "AT (J), AND KP=2:QD,WD="
6156 PRINT Z$;QD,WD
6170 Z$ = "REVIEW CALCULATED CP="
6171 PRINT Z$;CP
6185 FOR M = 1 TO 3
6186 PRINT
6187 NEXT M
6188 PRINT "ENTER '1' TO CHANGE CP OR '2' TO RETAIN CP"
6189 INPUT ZZ%
6190 ON ZZ% GOTO 6200,6215
6200 INPUT "ENTER NEW CP=";CP
6203 REM IF CP < 0. THEN CP=(QD-QF)/(WE-WF)
6215 Z$ = "FOR USE IN SEARCH TO FOLLOW,CP="
6216 PRINT Z$;CP
6245 FOR I = 1 TO NV
6246 VB(I) = VE(I)
6247 NEXT I
6248 IF MS = 0 THEN 6249 ELSE 6260
6249 GOSUB 1105
6250 GOTO 6275
6260 FOR IS = 1 TO MS
6261 OK(IS) = OE(IS)
6262 NEXT IS
6263 GOSUB 1105
6275 Z$ = "RETURNING TO LINE SEARCH WITH TUNED CP AND NEW BASE POINT:"
6276 PRINT Z$
6277 GOSUB 1015
6278 GOSUB 8240
6290 NT = ND
6291 ND = 1
6292 GOSUB 3000
6293 FR = AD
6294 NN = 2
6295 ND = NT
6296 FB = FR
6297 GOTO 5120
6305 REM *END OF CP SELF-TUNING PROCESS*
7997 REM *** SUBROUTINE 8000: PRINTOUT ***
7999 REM * INPUT DATA PRINTOUT *
8000 Z$ = "AB,AE,AG,AL,AM,AW,AT,CA,CL,CF,JX,KX="
8001 PRINT Z$
8002 PRINT AB,AE,AG,AL,AM,AW,AT,CA,CL,CF,JX,KX
8003 PRINT
8015 Z$ = "KC,KG,MD,MN,MS,MX,ND,NE,NF,NG,NP,NR,NU,NV="
8016 PRINT Z$
8017 PRINT KC,KG,MD,MN,MS,MX,ND,NE,NF,NG,NP,NR,NU,NV
8018 PRINT
8030 Z$ = "CN(I) FOR I=1,...,NV ARE="
8031 PRINT Z$
8032 FOR I = 1 TO NV

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```

8033 PRINT CN(I),
8034 NEXT I
8036 PRINT
8045 Z$ = "CX(I) FOR I=1,...,NV ARE="
8046 PRINT Z$
8047 FOR I = 1 TO NV
8048 PRINT CX(I),
8049 NEXT I
8050 PRINT
8060 IF NG = 0 THEN 8090 ELSE 8075
8075 Z$ = "CG(IG) FOR IG=1,...,NG ARE="
8076 PRINT Z$
8077 FOR IG = 1 TO NG
8078 PRINT CG(IG),
8079 NEXT IG
8080 PRINT
8090 IF MX > 0 THEN 8150 ELSE 8105
8105 Z$ = "SPECIFIED CS(I) FOR I=1,...,NV ARE="
8106 PRINT Z$
8107 FOR I = 1 TO NV
8108 PRINT CS(I),
8109 NEXT I
8110 PRINT
8120 IF MS = 0 THEN 8150 ELSE 8135
8135 Z$ = "SPECIFIED OS(IS) FOR IS=1,...,MS ARE="
8136 PRINT Z$
8137 FOR IS = 1 TO MS
8138 PRINT OS(IS),
8139 NEXT IS
8140 PRINT
8150 IF KC = 0 THEN 8210 ELSE 8165
8165 Z$ = "CALCULATED CONSTANTS C(IC) FOR IC=1,...,KC ARE="
8166 PRINT Z$
8167 FOR IC = 1 TO KC
8168 PRINT C(IC),
8169 NEXT IC
8170 PRINT
8171 GOTO 8210
8180 REM *NP=1,MINIMAL BASE POINT PRINTOUT OPTION*
8195 Z$ = "KT,VB(1),XB,QB="
8196 PRINT
8197 PRINT Z$
8198 PRINT KT,VB(1),XB,QB
8199 PRINT
8210 GOSUB 1015
8211 GOSUB 1030
8212 RETURN
8225 REM *NP=2,FULL BASE POINT PRINTOUT OPTION*
8240 Z$ = "KT,K,XB,QB="
8241 PRINT
8242 PRINT Z$
8243 PRINT KT,K,XB,QB
8244 PRINT
8255 Z$ = "VB(I) FOR I=1,...,NV ARE="
8256 PRINT Z$
8257 FOR I = 1 TO NV
8258 PRINT VB(I),
8259 NEXT I
8260 PRINT
8270 IF MS = 0 THEN 8315
8285 Z$ = "OK(IS) FOR IS=1,...,MS ARE="
8286 PRINT Z$
8287 FOR IS = 1 TO MS

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```

8288 PRINT OK(IS),
8289 NEXT IS
8290 PRINT
8300 Z$ = "SK(IS) FOR IS=1,...,MS ARE="
8301 PRINT Z$
8302 FOR IS = 1 TO MS
8303 PRINT SK(IS),
8304 NEXT IS
8305 PRINT
8315 IF NU > 0 THEN 8316 ELSE 8330
8316 Z$ = "UB(IU) FOR IU=1,...,NU ARE="
8317 PRINT Z$
8318 FOR IU = 1 TO NU
8319 PRINT UB(IU),
8320 NEXT IU
8321 PRINT
8330 IF NE > 0 THEN 8331 ELSE 8345
8331 Z$ = "EB(IE) FOR IE=1,...,NE ARE="
8332 PRINT Z$
8333 FOR IE = 1 TO NE
8334 PRINT EB(IE),
8335 NEXT IE
8336 PRINT
8345 IF NR > 0 THEN 8346 ELSE 8360
8346 Z$ = "RB(IR) FOR IR=1,...,NR ARE="
8347 PRINT Z$
8348 FOR IR = 1 TO NR
8349 PRINT RB(IR),
8350 NEXT IR
8351 PRINT
8360 IF KG > 0 THEN 8361 ELSE 8390
8361 Z$ = "NB(JG) FOR JG=1,...,KG ARE="
8362 PRINT Z$
8363 FOR JG = 1 TO KG
8364 PRINT NB(JG),
8365 NEXT JG
8366 PRINT
8390 GOSUB 1015
8391 GOSUB 1030
8392 RETURN
8496 REM ***SUBROUTINE 8500:INPUT DATA AND CALCULATED CONSTANTS***
8498 REM **INACTIVATE ANY 'DATA' NOT IN USE,BY DATA**
8500 Z$ = "INPUT DATA:"
8501 PRINT Z$
8502 PRINT
8505 AP = 3.141593
8506 AR = 57.29578
8507 CP = 0.
8508 KP = 0
8510 READ AB,AE,AG,AL,AM,AW,AT,CA,CL,CF,JX,KX
8512 DATA 10.,.5,1.E03,1.E05,8.,2.E04,2.,.85,.25,.25,100,300
8520 READ KC,KG,MD,MN,MS,MX,ND,NE,NF,NG,NP,NR,NU,NV
8529 GOSUB 1045
8530 REM FOR ARRAY DIMENSIONING
8531 FOR I = 1 TO NV
8532 READ CN(I)
8533 NEXT I
8540 FOR I = 1 TO NV
8541 READ CX(I)
8542 NEXT I
8549 IF NG = 0 THEN 8554
8550 FOR IG = 1 TO NG
8551 READ CG(IG)

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```

8552 NEXT IG
8553 REM DATA
8554 IF MX > 0 THEN 8560
8555 FOR I = 1 TO NV
8556 READ CS(I)
8557 NEXT I
8558 REM **SPECIFIED START POINT DATA (CASE OF NO SG)**
8560 IF MS = 0 THEN 8565
8561 FOR IS = 1 TO MS
8562 READ OS(IS)
8563 NEXT IS
8564 REM DATA
8565 IF KC = 0 THEN 8985
8566 REM ***HERE USER PROGRAMS EQUATIONS FOR CALCULATED
      REM      CONSTANTS C(IC), FOR IC=1 TO KC***
8567 REM *** SEE SUBROUTINE 18570 LINES 18571 - 18998 ARE AVAILABLE *
8568 REM **
8569 REM *
8570 GOSUB 18570
8982 REM *
8983 REM **
8984 REM ***
8985 GOSUB 8000
8986 RETURN
8992 REM ***SUBROUTINE 9000:ANALYSIS AT GIVEN V POINT***
8994 REM **HERE USER PROGRAMS EQUATIONS FOR:(1)U(IU),FOR
      REM      IU=1 TO NU; (2) N(JG),FOR JG=1 TO KG; (3) Q;
      REM      (4) E(IE),FOR IE=1 TO NE; (5) R(IR),FOR IR=1 TO NR**
8996 REM * USE MODSER REFORMULATION STRATEGY FOR THESE
      REM      EQUATIONS AND DECISION-MAKING LOGIC PROGRAMMED HERE *
8997 REM ***USE L COUNTER IF NECESSARY, INSTEAD OF I *
8998 REM ** SEE SUBROUTINE 19000 LINES 19001 - 19998 ARE AVAILABLE *
8999 REM *
9000 GOSUB 19000
9946 REM *
9947 REM **
9948 REM ***
9949 REM *** DO NOT ALTER LINES 9950 - 9993
9950 B = 0.
9951 H = 0.
9952 IF NE = 0 THEN 9970
9960 FOR IE = 1 TO NE
9961 B = B + E(IE) * E(IE)
9962 NEXT IE
9970 IF NR = 0 THEN 9990
9980 FOR IR = 1 TO NR
9981 IF R(IR) < 0. THEN H = H + R(IR) * R(IR)
9982 NEXT IR
9990 W = B + H
9991 P = CP * FP * W
9992 X = Q + P
9993 RETURN
9994 REM ***SUBROUTINE 10000:FINAL ITEMS ***
9996 REM ** HERE USER PROGRAMS FINAL ITEMS OF INTEREST,
      REM      BY F(JF) EQUATIONS,FOR JF=1 TO NF **
9998 REM** SEE SUBROUTINE 20010 LINES 20011 - 29999 ARE AVAILABLE *
10000 IF NF = 0 THEN RETURN
10001 GOSUB 1015
10002 ZF$ = "FINAL ITEMS F(JF) FOR JF=1,...,NF ARE="
10003 PRINT ZF$
10007 REM ***
10008 REM **
10009 REM *

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```
10010 GOSUB 20010
10989 REM *
10990 REM **
10991 REM ***
10992 FOR JF = 1 TO NF
10993 PRINT F(JF),
10994 NEXT JF
10995 RETURN
```

APPENDIX II

```

REM      PROGRAM      : Subroutine for the car suspension system
REM
REM      without damped absorber.
REM
REM      WRITTEN BY    : Rifaquat Cheema

18522    DATA 2.,0.,0.,0.,0.,0.,2.,0.,5.,6,1.,5,0.,2
18532    DATA 0.001,0.001
18542    DATA 1.,1.
18552    DATA 0.05,.1,1...001,1.,.001
18562    DATA 0.05,.5
18564    REM DATA
18565    RETURN
18570    DATA
18571    C(1)=CG(3)/CG(4)
18572    C(2)=CG(5)/CG(6)
18573    RETURN
19000    DATA
19001    A3=1.
19002    A2=2.*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))*(1.+1/CG(1))
19003    A1=CG(3)*V(1)+CG(3)*V(1)/CG(1)
19004    A0=0.
19007    D4=1.0
19008    D3=2.*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))*(1.+1./CG(1))
19009    D2=CG(3)*V(1)+1/CG(1)+CG(3)*V(1)/CG(1)
19010    D1=2.*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))/CG(1)
19012    D0=CG(3)*V(1)/CG(1)
19013    B3=0.
19014    B2=2*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))/CG(1)
19015    B1=CG(3)*V(1)/CG(1)
19016    B0=0.
19017    AI6=((A0**2/D0)*(D2*D3-D1*D4)+D3*(A1**2-2*A0*A2)+&
D1*(A2**2-2*A1*A3)+(A3**2/D4)*(D1*D2-D0*D3))/(&
D1*(D2*D3-D1*D4)-D0*D3**2)
19018    BI6=((B0**2/D0)*(D2*D3-D1*D4)&
+D3*(B1**2-2*B0*B2)+D1*(B2**2-2*B1*B3)+&
(B3**2/D4)*(D1*D2-D0*D3))/(D1*(D2*D3-D1*D4)-&
D0*D3**2)
19019    ACC=SQR(AP*ABS(BI6))
19021    Q=SQR(AP*ABS(AI6))
19023    R(1)=1.-V(1)
19024    R(2)=V(1)*C(1)-1.
19025    R(3)=1.-V(2)
19026    R(4)=V(2)*C(2)-1.
19027    R(5)=1.-ACC/CG(2)
19999    RETURN
20010    F(1)=Q
20011    F(2)=CG(3)*V(1)
20012    F(3)=CG(5)*V(2)
20015    F(4)=CG(1)
20016    F(5)=CG(2)
30000    RETURN
32000    END

```

```

REM      PROGRAM      : Subroutine for the car suspension system
REM
REM      when the damped absorber is attached to
REM
REM      sprung mass.
REM
REM      WRITTEN BY    : Rifaquat Cheema
REM
18522 DATA 4.,0.,0.,0.,0.,0.,2.,0.,8.,11.,2.,9,0.,4.
18532 DATA 0.001,0.001,0.001,0.001
18542 DATA 1.,1.,1.,1.
18552 DATA 0.05,.01,1.,.001,1.,.001,1.,.001,1.,.001,.3
18562 DATA 0.05,0.05,0.5,0.05
18564 REM DATA
18565 RETURN
18570 DATA
18571 C(1)=CG(3)/CG(4)
18572 C(2)=CG(5)/CG(6)
18573 C(3)=CG(7)/CG(8)
18574 C(4)=CG(9)/CG(10)
18575 RETURN
19000 DATA
19001 A5=1.
19002 A4=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*(1.+1./CG(2))+&
2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*(1.+1./CG(1))
19003 A3=4.*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*&
SQR(ABS(CG(5)*V(2)))*(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2))&
+CG(5)*V(2)*(1.+1./CG(2))+CG(3)*V(1)*(1.+1./CG(1))
19004 A2=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*CG(5)*V(2)*&
(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2))&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)*&
(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2)))
19005 A1=CG(3)*V(1)*CG(5)*V(2)*(1./CG(1)+1./CG(2)+1./(CG(2)*CG(1)))
19006 A0=0.0
19007 D6=1.0
19008 D5=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*(1.+1./CG(1))&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*(&
1.+1./CG(2))
19009 D4=1./CG(1)+CG(3)*V(1)*(1.+1./CG(1))+CG(5)*V(2)*&
(1.+1./CG(2))+4.*CG(7)*V(3)*&
*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(CG(5)*V(2)))*&
(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2)))
19010 D3=2.*CG(9)*V(4)*CG(3)*V(1)*SQR(ABS(CG(5)*V(2)))*&
(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2))&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*&
(1./(CG(1)*CG(2))+1./CG(1))+2*CG(7)*V(3)*CG(5)*V(2)*&
SQR(ABS(CG(3)*V(1)))*(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2))&
+2*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*1./CG(1)
19011 D2=CG(3)*V(1)*CG(5)*V(2)*(1./CG(1)+1./CG(2)+1./(CG(1)*CG(2))&
+CG(5)*V(2)*(1./CG(1)+1./(CG(1)*CG(2))+CG(3)*V(1)/CG(1)+&
4*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(&
CG(5)*V(2)))/(CG(1)*CG(2))
19012 D1=(2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)+&
2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*CG(5)*V(2))*&
1./(CG(1)*CG(2))
19013 D0=(CG(3)*V(1)*CG(5)*V(2))/(CG(1)*CG(2))
19014 Q1=-(D0*D1*D5)+D0*D3**2-D1*D2*D3+D1**2*D4
19015 Q2=D0*D3*D5+(D1**2)*D6-D1*D2*D5
19016 Q3=D0*D5**2+D1*D3*D6-D1*D4*D5
19017 Q4=1./D0*(D2*Q3-D4*Q2+D6*Q1)
19018 Q5=1./D0*(D2*Q4-D4*Q3+D6*Q2)
19019 Q0=1./D6*(D4*Q1-D2*Q2+D0*Q3)
19020 P6=D0*(D1*Q5-D3*Q4+D5*Q3)

```

2.*A1*A5)*Q2+Q3*(A2**2-2*A1*A3+2.*A0*A4)+Q4*(A1**2-2.*A0*A2)&
+A0**2*Q5)

```
19022 B5=0.
19023 B4=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))/CG(1)
19024 B3=4*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*&
SQR(ABS(CG(5)*V(2)))/(CG(1)*CG(2))+CG(3)*V(1)/CG(1)
19025 B2=(2.*CG(7)*V(3)*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))+&
2.*(CG(9)*V(4)*CG(3)*V(1)*SQR(ABS(CG(5)*V(2))))/&
(CG(1)*CG(2))
19026 B1=(CG(3)*V(1)*CG(5)*V(2))/(CG(1)*CG(2))
19027 B0=0.
19028 BI6=1./(2.*P6)*(B5**2*Q0+Q1*(B4**2-2.*B3*B5)+(B3**2-2.*B2*B4+&
2.*B1*B5)*Q2+Q3*(B2**2-2*B1*B3+2.*B0*B4)+Q4*(B1**2-2.*B0*B2)&
+B0**2*Q5)
19029 ACC=SQR(2.*AP*ABS(BI6))
19030 Q=SQR(2.*AP*ABS(AI6))
19031 R(1)=1.-V(1)
19032 R(2)=V(1)*C(1)-1.
19033 R(3)=1.-V(2)
19034 R(4)=V(2)*C(2)-1.
19035 R(5)=1.-V(3)
19036 R(6)=V(3)*C(3)-1.
19037 R(7)=1.-V(4)
19038 R(8)=V(4)*C(4)-1.
19039 R(9)=1.-ACC/CG(11)
19999 RETURN
20010 F(1)=Q
20011 F(2)=CG(3)*V(1)
20012 F(3)=CG(5)*V(2)
20013 F(4)=CG(7)*V(3)
20014 F(5)=CG(9)*V(4)
20015 F(6)=CG(1)
20016 F(7)=CG(2)
20017 F(8)=CG(11)
30000 RETURN
32000 END
```

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REM      PROGRAM      : Subroutine for the car suspension system
REM
REM      when the damped absorber is attached to
REM
REM      unsprung mass.
REM
REM      WRITTRN BY      : Rifaquat Cheema
REM
18522    DATA 4.,0.,0.,0.,0.,0.,2.,0.,8.,11.,1.,9.,0.,4.
18532    DATA 0.001,0.001,0.001,0.001
18542    DATA 1.,1.,1.,1.
18552    DATA 0.05,.01,1.,.001,1.,.001,1.,.001,1.,.001,.3
18562    DATA 0.05,0.05,0.5,0.05
18564    REM DATA
18565    RETURN
18570    DATA
18572    C(1)=CG(3)/CG(4)
18573    C(2)=CG(5)/CG(6)
18574    C(3)=CG(7)/CG(8)
18575    C(4)=CG(9)/CG(10)
18576    RETURN
19000    DATA
19001    A5=1.
19002    A4=2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*(1/CG(1)+1./CG(2))+&
2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*(1.+1/CG(1))
19003    A3=4.*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*&
SQR(ABS(CG(5)*V(2)))*(1/CG(1)+1/CG(2)+1/(CG(1)*CG(2)))*&
+CG(5)*V(2)*(1/CG(1)+1/CG(2))+CG(3)*V(1)*(1+1/CG(1))
19004    A2=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*CG(5)*V(2)*&
(1/CG(1)+1/CG(2)+1/(CG(1)*CG(2)))*&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)*&
(1/CG(1)+1/CG(2)+1/(CG(1)*CG(2)))
19005    A1=CG(3)*V(1)*CG(5)*V(2)*(1/CG(1)+1/CG(2)+1/(CG(2)*CG(1)))
19006    A0=0.0
19007    D6=1.0
19008    D5=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*(1.+1./CG(1))*&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*1./CG(1)+&
2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*1./CG(2)
19009    D4=1./CG(1)+CG(3)*V(1)*(1.+1./CG(1))+CG(5)*V(2)*&
(1./CG(1)+1./CG(2))+4.*CG(7)*V(3)*&
*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(CG(5)*V(2)))*&
(1./CG(1)+1./CG(2)+1/(CG(1)*CG(2)))
19010    D3=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*&
(1./CG(1)+CG(5)*V(2)*(1/(CG(1)*CG(2))+1./CG(1)+1./CG(2)))*&
+2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*&
(1./(CG(1)*CG(2))+CG(3)*V(1)*(&
1/CG(1)+1/CG(2)+1/(CG(1)*CG(2))))
19011    D2=CG(3)*V(1)*CG(5)*V(2)*(1/CG(1)+1/CG(2)+1/(CG(1)*CG(2)))*&
+CG(5)*V(2)*(1/(CG(1)*CG(2))+CG(3)*V(1)/CG(1)+&
4*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*SQR(ABS(&
CG(5)*V(2)))/(CG(1)*CG(2)))
19012    D1=(2.*CG(9)*V(4)*SQR(ABS(CG(5)*V(2)))*CG(3)*V(1)+&
2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))*CG(5)*V(2))*&
1/(CG(1)*CG(2))
19013    D0=(CG(3)*V(1)*CG(5)*V(2))/(CG(1)*CG(2))
19014    Q1=-(D0*D1*D5)+D0*D3**2-D1*D2*D3+D1**2*D4
19015    Q2=D0*D3*D5+(D1**2)*D6-D1*D2*D5
19016    Q3=D0*D5**2+D1*D3*D6-D1*D4*D5
19017    Q4=1./D0*(D2*Q3-D4*Q2+D6*Q1)
19018    Q5=1./D0*(D2*Q4-D4*Q3+D6*Q2)
19019    Q0=1./D6*(D4*Q1-D2*Q2+D0*Q3)
19020    P6=D0*(D1*Q5-D3*Q4+D5*Q3)
19021    AI6=1./(2.*P6)*(A5**2*Q0+Q1*(A4**2-2.*A3*A5)+(A3**2-2.*A2*A4+&

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2.*A1*A5)*Q2+Q3*(A2**2-2*A1*A3+2.*A0*A4)+Q4*(A1**2-2.*A0*A2)&
+A0**2*Q5)
19022 B5=0.
19023 B4=2.*CG(7)*V(3)*SQR(ABS(CG(3)*V(1)))/CG(1)
19024 B3=4*CG(7)*V(3)*CG(9)*V(4)*SQR(ABS(CG(3)*V(1)))*&
SQR(ABS(CG(5)*V(2)))/(CG(1)*CG(2))+CG(3)*V(1)/CG(1)
19025 B2=(2.*CG(7)*V(3)*CG(5)*V(2)*SQR(ABS(CG(3)*V(1)))+&
2.*(CG(9)*V(4)*CG(3)*V(1)*SQR(ABS(CG(5)*V(2))))/&
(CG(1)*CG(2))
19026 B1=(CG(3)*V(1)*CG(5)*V(2))/(CG(1)*CG(2))
19027 B0=0.
19028 BI6=1./((2.*P6)*(B5**2*Q0+Q1*(B4**2-2.*B3*B5)+(B3**2-2.*B2*B4+&
2.*B1*B5)*Q2+Q3*(B2**2-2*B1*B3+2.*B0*B4)+Q4*(B1**2-2.*B0*B2)&
+B0**2*Q5)
19029 ACC=SQR(2.*AP*ABS(BI6))
19030 Q=SQR(2.*AP*ABS(AI6))
19031 R(1)=1.-V(1)
19032 R(2)=V(1)*C(1)-1.
19033 R(3)=1.-V(2)
19034 R(4)=V(2)*C(2)-1.
19035 R(5)=1.-V(3)
19036 R(6)=V(3)*C(3)-1.
19037 R(7)=1.-V(4)
19038 R(8)=V(4)*C(4)-1.
19039 R(9)=1-ACC/CG(11)
19999 RETURN
20010 F(1)=Q
20011 F(2)=CG(3)*V(1)
20012 F(3)=CG(5)*V(2)
20013 F(4)=CG(7)*V(3)
20014 F(5)=CG(9)*V(4)
20015 F(6)=CG(1)
20016 F(7)=CG(2)
20017 F(8)=CG(11)
30000 RETURN
32000 END

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